Mathematics Mastery

Year 9 Module Overviews

We are guided by four underlying principles:

- high expectations for every child – one curriculum
- depth before breadth
- number sense and place value come first
- problem solving at the heart

First, we agree that mathematical intelligence is expandable. In a sense, if we didn't believe this, we wouldn't be teaching. But success in mathematics often seems to be used as an indicator of 'innate' intelligence, rather than something that everyone can achieve with effort. We believe that every child can learn mathematics, given the appropriate learning experiences within and beyond the classroom. We therefore have a responsibility to map our curriculum to enable every child to succeed. Our curriculum map reflects our **high expectations for every child.** Every student is entitled to master the key mathematical content for their age. Every student must receive the support and challenge they need. We believe that this personalisation can be achieved with all students learning the same concepts and skills.

The second thing we agree on is the importance of deep progress. National Curriculum level descriptors have led us to equate progress with knowing new procedures and rules. Many students build a superficial knowledge of a large number of techniques, but find that at GCSE, A level or beyond they lack the depth of understanding to be able to use these skills. We focus on fewer key concepts in each term, putting **depth before breadth,** and students demonstrate progress by making connections between representations, and applying them within and beyond the curriculum. This structure liberates. Teachers find that spending longer on each topic enables them to really think and talk about the mathematics they are teaching. The curriculum is cumulative. We sequence the concepts and methods so that previously learnt ideas can be connected to new learning, supporting students in understanding the coherent and connected nature of the subject, and ensuring they consolidate learning by continually using and applying it in a variety of contexts.

Mathematics is a rich and varied subject, and throughout primary and secondary education students experience a wealth of concepts and skills, including algebra, geometry and statistics. We believe that all of mathematics can be appreciated more fully once a student has a deep appreciation of the number system, and therefore we put **number sense and place value first**.

Problem solving is at the heart of mathematics. We structure our curriculum so that all students in a year group learn the same content at the same time, have longer to focus on this content, and spend a significant amount of time securing essential number skills. In this way we aim to create the optimal conditions for students to both learn through problem solving and to learn to solve problems.

How to use these unit overviews

These unit overviews are designed to be used by teachers in schools that are members of the Mathematics Mastery community. They should be interpreted by experienced teachers and leaders within the context of the philosophy, aims, curricula and pedagogical principles of the mastery approach. A very few pertinent features are re-emphasised here, but this alone is not sufficient for the approach to be effectively interpreted.

Mastery objectives are cumulative. At the end of the year, students should know, understand and be able to do *every* objective included here. Objectives specified for a unit should not only be considered to be the learning for an individual lesson or discrete series of lessons, but rather be explicitly taught during the specified unit, and then applied in future lessons as well as in other areas of the curriculum and beyond. This applies both within and across half terms. When a concept or skill is first introduced for the key stage, it is highlighted in grey.

Mathematics Mastery

Year 9 Module 1 (Autumn 1) Graphs and Proportion

This half term, students will:

Working mathematically

Develop fluency

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- substitute values in expressions, rearrange and simplify expressions, and solve equations
- move freely between different numerical, algebraic, graphical and diagrammatic representations
- develop algebraic and graphical fluency, including understanding linear functions
- use language and properties precisely to analyse numbers and algebraic expressions

Reason mathematically

- extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
- extend and formalise their knowledge of ratio and proportion in formulating proportional relations algebraically
- identify variables and express relations between variables algebraically and graphically
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- begin to reason deductively in number and algebra
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning

Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

Subject content

Number

- recognise and use relationships between operations including inverse operations
- use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations
- interpret and compare numbers in standard form $A \times 10^n$ $1 \leq A < 10$, where n is a positive or negative integer or zero
- work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately

Algebra

- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, formulae inequalities, terms and factors
- model situations or procedures by translating them into algebraic expressions
- use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)

- work with coordinates in all four quadrants
- recognise, sketch and produce graphs of linear functions of one variable with appropriate scaling, using equations in x and y and the Cartesian plane
- interpret mathematical relationships both algebraically and graphically
- use a given linear equation in two variables in the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically
- use linear graphs to estimate values of y for given values of x and vice versa
- find approximate solutions to contextual problems from given graphs of a variety of functions, including linear graphs

Ratio, proportion and rates of change

- use scale factors, scale diagrams and maps
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions
- solve problems involving direct and inverse proportion, including graphical and algebraic representations

Geometry and measures

- calculate and solve problems involving: perimeters of 2-D shapes
- draw and measure line segments and angles in geometric figures, including interpreting scale drawings
- derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies
- interpret mathematical relationships both algebraically and geometrically.

Unit 1 **Coordinates** (1 week)	Within this week's unit, students will learn to: • Plot coordinates in all four quadrants • Apply their knowledge of 2D shapes to coordinate problems • Find the midpoint of a line segment joining two points • Find an endpoint of a line segment, given the midpoint and one endpoint • Solve problems using coordinate grids Students will be familiar with coordinates from work at primary school and in other subjects. This unit provides opportunities for students to solve problems in all four quadrants, reinforcing their knowledge of negative numbers and properties of triangles and quadrilaterals. They should understand why the midpoint of a line segment is the "mean of the coordinates" and devise strategies for solving related problems.
Unit 2 **Linear Graphs** (2 weeks)	Within this fortnight's unit, students will learn to: • Identify the equations of horizontal and vertical lines • Plot coordinates from a rule to generate a straight line • Identify key features of a linear graph • Make links between the graphical and the algebraic representation • Identify parallel lines from algebraic equations This unit builds on the previous one and is students' first formal introduction to straight line graphs. Students should be encouraged to see the links/similarities/differences between the different mathematical representations of a relationship e.g. the equation, the coordinates and the graphical representation. Using real contexts will help assign practical meaning to the gradient and the intercept. Parallel and perpendicular lines provide more opportunities for revisiting shape work. Optional extension activities with 3D coordinates are also provided.

Unit 3 ***Direct and Inverse proportion*** (2 weeks)	Within this fortnight's unit, students will learn to: • Interpret and analyse real-life linear graphs • Consider the applications of linear graphs to real-life problems • Recognise when two quantities are directly proportional to each other • Recognise when two quantities are inversely proportional to each other • Recognise the graphical representation of a proportional relationship • Solve proportion problems This unit builds on students' previous work on ratio (Year 8 Unit 8) and linear graphs (above) including by linking alternate representations of the same relationship and using these to solve problems. Inverse proportion is introduced by consideration of everyday applications such as speed and time.
Unit 4 ***Scales and Standard Form*** (1 week)	Within this week's unit, students will learn to: • Use standard form to express very large and small numbers • Convert between standard form and ordinary numbers • Order large and small numbers • Use standard form to solve simple problems • Use scales to solve distance and area problems in context Students will use calculators to explore and understand scientific notation. Links will be made to indices work in Year 8 Unit 2 and scale factors (above) as powers of ten will be used to develop strategies to solve scaling problems.

Mathematics Mastery

Year 9 Module 2 (Autumn 2) Algebraic Expressions

This half term, students will:

Working mathematically

Develop fluency

- select and use appropriate calculation strategies to solve increasingly complex problems
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- substitute values in expressions, rearrange and simplify expressions, and solve equations
- move freely between different numerical, algebraic, graphical and diagrammatic representations
- develop algebraic and graphical fluency, including understanding linear functions
- use language and properties precisely to analyse numbers and algebraic expressions

Reason mathematically

- extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
- extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically
- identify variables and express relations between variables algebraically and graphically
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- begin to reason deductively in number and algebra

Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

Subject content

Number

- use conventional notation for the priority of operations, including brackets, powers, roots and reciprocals
- recognise and use relationships between operations including inverse operations
- appreciate the infinite nature of the sets of integers, real and rational numbers.

Algebra

- use and interpret algebraic notation, including:
  - ab in place of $a \times b$
  - $3y$ in place of $y + y + y$ and $3 \times y$
  - a^2 in place of $a \times a$
  - $\frac{a}{b}$ in place of $a \div b$
  - coefficients written as fractions rather than as decimals
  - brackets
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, formulae, inequalities, terms and factors
- simplify and manipulate algebraic expressions to maintain equivalence by
  - collecting like terms
  - multiplying a single term over a bracket
  - taking out common factors
  - expanding products of two or more binomials
- understand and use standard mathematical formulae; rearrange formulae to change the subject

	• model situations or procedures by translating them into algebraic expressions • interpret mathematical relationships both algebraically and graphically • reduce a given linear equation in two variables to the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically • generate terms of a sequence from either a term-to-term or a position-to-term rule • recognise arithmetic sequences and find the n^{th} term • recognise geometric sequences and appreciate other sequences that arise.
Unit 5 **Sequences** (1 week)	Within this week's unit, students will learn to: • Recognise linear sequences and non-linear sequences • Find the rule for the n^{th} term for a linear sequence • Generate sequences from n^{th} term formulae • Explore simple non-linear sequences • Generate sequences from a given context • Solve problems involving a variety of sequences Students will have met sequences in Year 7 and Year 8. In this unit, opportunity to consolidate work on finding the n^{th} term in a variety of contexts. Students are also introduced to non-linear sequences such as square numbers, triangular numbers and powers of 2. Students should be supported to recognise alternative representations of the same sequence e.g. pictorial, algebraic and graphical.
Unit 6 **Expanding and Factorising** (2 weeks)	Within this fortnight's unit, students will learn to: • Multiply a term over a single bracket • Expand products of two or more binomials • Make links between area and perimeter and expanding brackets • Factorise expressions into a single bracket • Factorise quadratic expressions where the coefficient of x^2 is equal to one Students will consolidate their knowledge of area and perimeter in using these as a vehicle for exploring expansion of brackets. They will be supported to appreciate how general expressions relate to specific numerical cases.
Unit 7 **Changing the subject of a formula** (2 weeks)	Within this fortnight's unit, students will learn to: • Write expressions, equations and formulae to represent relationships in a given context • Use informal substitution to find the value of one variable given other values • Make links between solving linear equations and rearranging formulae • Apply "changing the subject" to equations of straight lines, including to identify parallel lines • Manipulate familiar formulae such as known formulae for area and perimeter Students will explore manipulating equations and formulae in a variety of practical contexts. There is opportunity here to consolidate earlier work on linear equations, linear graphs and geometrical formulae that have already been met. The focus is on working in familiar contexts rather than manipulating abstract expressions, thus allowing students to make connections and develop their intuition.

Mathematics Mastery

Year 9 Module 3 (Spring 1) 2D Geometry

This half term, students will:

Working mathematically

Develop fluency

- select and use appropriate calculation strategies to solve increasingly complex problems

Reason mathematically

- extend and formalise their knowledge of ratio and proportion in working with measures and geometry
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- begin to reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning

Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

Subject content

Number

- work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$or 0.375 and $\frac{3}{8}$)
- interpret fractions and percentages as operators
- use standard units of mass, length, time, money and other measures, including with decimal quantities
- round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]

Ratio, proportion and rates of change

- use scale factors
- express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1
- use ratio notation, including reduction to simplest form
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions
- solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics

Geometry and measures

- draw and measure line segments and angles in geometric figures, including interpreting scale drawings
- derive and use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); recognise and use the perpendicular distance from a point to a line as the shortest distance to the line
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively and rotationally symmetric

- use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the criteria for congruence of triangles
- derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies
- identify and construct congruent triangles, and construct similar shapes by enlargement, with and without coordinate grids
- derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons
- interpret mathematical relationships both algebraically and geometrically.

Note: There is a large overlap between the content of Units 8, 9 and 10 and there will inevitably be some crossover. When planning your lessons from our resource bank, please look at the content of all the units and adapt/mix according to your students' needs.

Unit 8
Constructions
(1 weeks)
	Within this week's unit, students will learn to:
	• Use the standard ruler and compass constructions for:
	  ○ perpendicular bisector of a line segment
	  ○ constructing a perpendicular to a given line from/at a given point
	  ○ bisecting a given angle
	• Understand and use the perpendicular distance from a point to a line as the shortest distance to the line
	• Construct triangles and quadrilaterals from given information
	• Construct regular polygons within circles
	The shapes that students construct in this unit will be used throughout the remainder of this module to support students' understanding of geometrical concepts and relationships, in particular congruency and similarity. They will work primarily with rulers and pairs of compasses, but there will be some opportunity to revisit protractor work when checking the accuracy of angle bisection and in constructing triangles. Students will also develop their knowledge of the correct notation for labelling/naming angles and sides within shapes and the conventions for parallel sides, etc.

Unit 9 **Congruence and Similarity** (2 weeks)	Within this fortnight's unit, students will learn to: • Derive and use the conditions for congruent triangles • Appreciate when any two shapes are congruent • Enlarge shapes from a given centre, with and without coordinate grids • Understand the difference between congruence and similarity • Understand that the corresponding angles of similar shapes are equal • Find missing sides in similar shapes • Solve problems involving similar triangles Students will again be able to use the constructions from Unit 8 above to discover the difference between similar and congruent shapes. This topic also provides good opportunities to revisit the ratio concepts from Year 8 Unit 8 and the scales work from Unit 4 of Year 9, considering and comparing different approaches to solving the same problem.

Unit 10 **Triangles and Quadrilaterals** (1 week)	Within this week's unit, students will learn to: • Appreciate the symmetry properties of triangles and special quadrilaterals • Investigate the properties of the diagonals of quadrilaterals and the angles formed when they cross This unit links very closely to the ones above and teachers may wish to make use of the shapes that students have recently constructed, for example by considering what information is needed to ensure two triangles are congruent. Students will be able to make links between the constructions they have learnt in the last unit and the diagonal properties of quadrilaterals and also investigate the congruency of the triangles formed when a quadrilateral's diagonals are joined. They will also revisit rotational and reflection symmetry.
Unit 11 ***Angles in Polygons*** (1 week)	Within this week's unit, students will learn to: • Derive the proof of the sum of the angles in a triangle • Find the formula for sum of the angles of any polygon • Understand and use the sum of the exterior angles of a polygon • Understand the difference between regular and irregular polygons • Solve problems involving the angles/number of sides in a regular polygon Students will revisit their angle facts from earlier work in both Years 7 and 8 to understand the proof of the sum of the angles in a triangle. They will then investigate how other polygons, starting with quadrilaterals, can be divided into triangles and so deduce the general formula. Following on from investigating exterior angles, they will then solve a series of problems such as finding the number of sides given information about the angles of polygons.

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Year 9 Module 4 (Spring 2) Equations and Inequalities

This half term, students will:

Working mathematically

Develop fluency

- select and use appropriate calculation strategies to solve increasingly complex problems
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- substitute values in expressions, rearrange and simplify expressions, and solve equations
- move freely between different numerical, algebraic, graphical and diagrammatic representations
- develop algebraic and graphical fluency, including understanding linear and simple quadratic functions
- use language and properties precisely to analyse numbers and algebraic expressions

Reason mathematically

- extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
- identify variables and express relations between variables algebraically and graphically
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- begin to reason deductively in geometry, number and algebra

Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

Subject content

Number

- use conventional notation for the priority of operations, including brackets, powers, roots and reciprocals
- recognise and use relationships between operations including inverse operations
- use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations

Algebra

- understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- simplify and manipulate algebraic expressions to maintain equivalence by
  - collecting like terms
  - multiplying a single term over a bracket
  - taking out common factors
- use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)
- work with coordinates in all four quadrants
- recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in x and y on the Cartesian plane
- interpret mathematical relationships both algebraically and graphically
- reduce a given linear equation in two variables to the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically
- use linear and quadratic graphs to estimate values of y for given values of x and vice versa and

<table>
<tr><td colspan="2">to find approximate solutions of simultaneous linear equations
• find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, exponential and reciprocal graphs</td></tr>
<tr><td>Unit 12
Linear equations and inequalities
(2 weeks)</td><td>Within this fortnight's unit, students will learn to:
• Form and solve linear equations in one unknown, including those where the unknown appears on both sides
• Rearrange and solve linear equations given in any form, including those including those involving fractions and brackets
• Express relationships using inequality notation
• Form and solve linear inequalities in one unknown, including those where the unknown appears on both sides.
Students will build on their skills from Year 8 Unit 4 to form and solve increasingly complex equations. There will be an emphasis on the formation of the equations from a given context. For solving the equations, the use of bar models and other representations such as balances will be encouraged. Students will then explore inequalities through contextual problems involving maximising and minimising. This unit also allows for revisiting and extending Unit 6 from earlier this year. Links should also be made to the graphical work of Unit 1 and Unit 2.</td></tr>
<tr><td>Unit 13
Simultaneous equations
(1 week)</td><td>Within this week's unit, students will learn to:
• Appreciate the links between the graphical and algebraic representations of equations
• Use graphs to find approximate solutions to linear simultaneous equations
• Understand that the accuracy of solutions can be checked through substitution into the original equations
Students will be introduced to two variable problems and appreciate that one equation is not enough to determine a single solution. They will reinforce their skills in drawing linear graphs, including some that will need rearrangement.</td></tr>
<tr><td>Unit 14
Quadratic and other Graphs
(2 weeks)</td><td>Within this two-week unit, students will learn to:
• Draw quadratic graphs
• Solve problems using given quadratic graphs
• Solve problems using given reciprocal graphs
• Solve problems using given piece-wise linear graphs
• Solve problems using given exponential graphs
Students will extend their experience from the previous unit to the drawing and finding of approximate solutions from quadratic graphs; substitution will enable them to see that solutions are far less likely to be exact. They will also be able to reinforce their work from Unit 4 through choosing scales for the graphs they construct and through the reading of scales for given graphs. They will then apply these skills to given graphs in a wide range of other situations.</td></tr>
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Year 9 Module 5 (Summer 1) Geometry

This half term, students will:

Working mathematically

Develop fluency

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- substitute values in expressions, rearrange and simplify expressions, and solve equations
- move freely between different numerical, algebraic, graphical and diagrammatic representations
- use language and properties precisely to analyse numbers, algebraic expressions, 2-D and 3-D shapes and probability.

Reason mathematically

- extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
- extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- begin to reason deductively in geometry, number and algebra, including using geometrical constructions
- explore what can and cannot be inferred in probabilistic settings, and begin to express their arguments formally.

Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

Subject content

Number

- recognise and use relationships between operations including inverse operations
- round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately
- use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor and lowest common multiple
- use the four operations, including formal written methods, applied to integers, decimals and proper fractions
- work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$or 0.375 and $\frac{3}{8}$)
- define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another and compare two quantities using percentages

Algebra

- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use standard mathematical formulae
- model situations or procedures by translating them into algebraic expressions

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Ratio, proportion and rates of change

- use scale factors, scale diagrams and maps
- express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- solve problems involving direct and inverse proportion, including graphical and algebraic representations

Geometry and measures

- recognise and use the perpendicular distance from a point to a line as the shortest distance to the line
- use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the criteria for congruence of triangles
- identify properties of, and describe the results of, translations, rotations and reflections applied to given figures
- identify and construct congruent triangles, and construct similar shapes by enlargement, with and without coordinate grids
- apply angle facts, triangle congruence, similarity and properties of quadrilaterals to derive results about angles and sides, including Pythagoras' Theorem, and use known results to obtain simple proofs
- use Pythagoras' Theorem and trigonometric ratios in similar triangles to solve problems involving right-angled triangles
- interpret mathematical relationships both algebraically and geometrically.

Probability

- record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale
- understand that the probabilities of all possible outcomes sum to 1
- enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams
- generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities.

Unit 15 **Pythagoras' theorem**	Within this week's unit, students will learn to: • Prove and use Pythagoras' theorem to find missing sides in right-angled triangles • Solve associated problems in other shapes including right-angled triangles • Deduce whether a triangle is right-angled by considering its sides Students will use both algebraic and geometric methods to prove the veracity of Pythagoras' theorem. They will then use the theorem in a variety of contexts to solve problems and find the lengths of any missing sides.
Unit 16 **Transformations** (2 weeks)	Within this two-week unit, students will learn to: • Translate a shape by a given vector • Reflect a shape in a line, including on coordinate axes • Rotate a shape about a centre, including on coordinate axes • Identify the type of transformation carried out by comparing an object and image Enlargement will have been covered during Unit 9. In this unit, students will explore the other standard types of transformations and consider which properties are invariant under which transformation. There will be a strong emphasis on the use of correct mathematical language and consideration of what detail is needed to effectively describe a transformation. This is also an opportunity to revisit properties of shapes.

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Unit 17 **Probability** (2 weeks)	Within this two-week unit, students will learn to: • Understand and use the probability scale from 0 to 1 • Understand and use the language associated with probability • Understand what is meant by "random" • Appreciate the difference between experimental and theoretical probability • Understand the relationship between relative frequency and theoretical probability • Understand that different trials of an experiment may well produce different outcomes • Systematically list outcomes using a variety of representations • Use Venn diagrams and understand the meaning of union and intersection It is important that students learn the numerical representation of probability alongside terms such as "likely" and "unlikely" rather than after them so they have some sense of what the words mean in mathematical context. Students will also be able to practise their skills in converting fractions, decimals and percentages and appreciate that the use of ratios to express probability can be misleading. They will learn to appreciate that repeating an experiment more often will tend to a more accurate estimate of a probability. Common misconceptions and misunderstandings of probability will be challenged.

Mathematics Mastery

Year 9 Module 6 (Summer 2) Statistics

This half term, students will:

Working mathematically

Develop fluency

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- move freely between different numerical, algebraic, graphical and diagrammatic representations
- use language and properties precisely to analyse numbers, algebraic expressions, 2-D and 3-D shapes, probability and statistics.

Reason mathematically

- identify variables and express relations between variables algebraically and graphically
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- explore what can and cannot be inferred in statistical and probabilistic settings, and begin to express their arguments formally.

Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

Subject content

Number

- use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor and lowest common multiple
- use the four operations, including formal written methods, applied to integers, decimals and proper fractions
- work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$or 0.375 and $\frac{3}{8}$)
- define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another and compare two quantities using percentages

Ratio, proportion and rates of change

- express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1

Statistics

- describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data; and appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)
- construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data
- describe simple mathematical relationships between two variables (bivariate data) in observational and experimental contexts and illustrate this using scatter graphs.

Unit 18 **Working with data** (1 week)	Within this week's unit, students will learn to: • Interpret and draw stem and leaf diagrams, appreciating the need for a key • Use a wide variety of representations and averages to compare a set of distributions • Appreciate the difference between discrete and continuous data • Understand why the exact mean cannot be found from grouped data • Find an estimate of the mean from grouped data and continuous data Students will revisit the mean average, first met in Year 7 Unit 7 and the mean of frequency distributions, met at the end of Year 8, to work with grouped data. They will look at both continuous and discrete distributions. The main focus of this unit will be interpretation, rather than construction, of statistical diagrams, although there will be opportunities to practise this if needed. As well as determining the usefulness of different types of diagram for making comparisons, students will need to consider which average is most appropriate to describe a distribution and whether using the range as a measure of spread is also appropriate. They will also revisit the collection of data and the dangers of bias.
Unit 19 ***Scatter graphs*** (1 week)	Within this week's unit, students will learn to: • Plot scatter graphs • Describe the type of correlation observed • Interpret correlation in the context of the data set Students will collect bivariate data and use the graphical representation to make simple inferences about the relationship. It would be particularly useful to make lines with science and other areas of the curriculum during this unit.
Unit 20 ***Introduction to trigonometry*** (2 weeks)	• Investigate the trigonometric ratios using similar triangles • Define and use the cosine, sine and tangent ratios • Use tables of trigonometric values to find missing sides or angles in right-angled triangles • Solve problems involving right-angled triangles using the trigonometric ratios Students will deepen their understanding of similarity (Unit 9) by considering the trigonometric ratios in right-angled triangles. This unit will provide preparation for the GCSE course through extending students' understanding of similarity. To emphasise that sine, cosine and tangent are ratios, the unit will exclusively use tables of trigonometric values, and will not introduce the calculator functions.