Year 7

Fractions of amounts

Within this **week's unit**, students will learn to:

- Express one quantity as a fraction of another
- Find a fraction of a set of objects or quantity

Mathematics Mastery

Resources available for this unit:

Ideas for representations

Further depth tasks

Coaching

Workbook

Departmental Workshop

Question prompts

L1: Can you share this between [...] groups? What fraction of the whole is each group? How do you know?
How can you show a sixth of (e.g.) 24?
How can equivalent fractions help?
How can you draw a bar model to show one quarter of 120? What do each of the parts of the bar represent?
What operation is helpful?
If you know a half how do you find a quarter? If you know a third how do you find a twelfth?

L2: [For 'fact families'] What is a factor pair? How is this relevant?
What links multiplication and division? How are these important for finding fractions of quantities?
If we can find one quarter, how can we find three quarters? Is there another way?
What is the same or different between the line and bar model?
How could you scale a number to be half / three quarters / five quarters as big?

L3: How many equal parts are there in the whole?
How many parts represent the known quantity?
How can we represent this information? Which representation is most helpful? Why?
How could we find the value of each equal part?
What might our next step be?
Which mathematical operations are useful?
Is it helpful to find the value of the whole first? Explain.
What other fractions can you find?
Which other fractions are easiest to find given: $\frac{3}{5}, \frac{2}{3}, \frac{5}{8}$, etc.?
Which part of the fraction do you consider first when solving these problems, the numerator or the denominator? Why?

L4: How can you represent this with a bar model?
What next steps could you take to solve the problem?
How do you calculate the area of a rectangle / triangle?
What happens to the area if you halve the base / width / etc.?
Can you find another way to solve this?
How is it the same or different to you original method?

L5: [For 'Do now'] How can you explain which is the odd one out? What key words do you need to explain?
[For 'True or False'] How can you prove that these are true or false? Would this be different if you changed the set of numbers? How?
What do you know about angles? How can you represent angles at a point / the internal angle sum with a bar model?
[For 'What's the perimeter?'] What do you know about the perimeter of a rectangle? How could you represent the full perimeter with a bar model? What do you know about each of the parts?

Thursday, 04 June 2020

Lesson 1

Calculating with unit fractions

Key learning

Find unit fractions of quantities

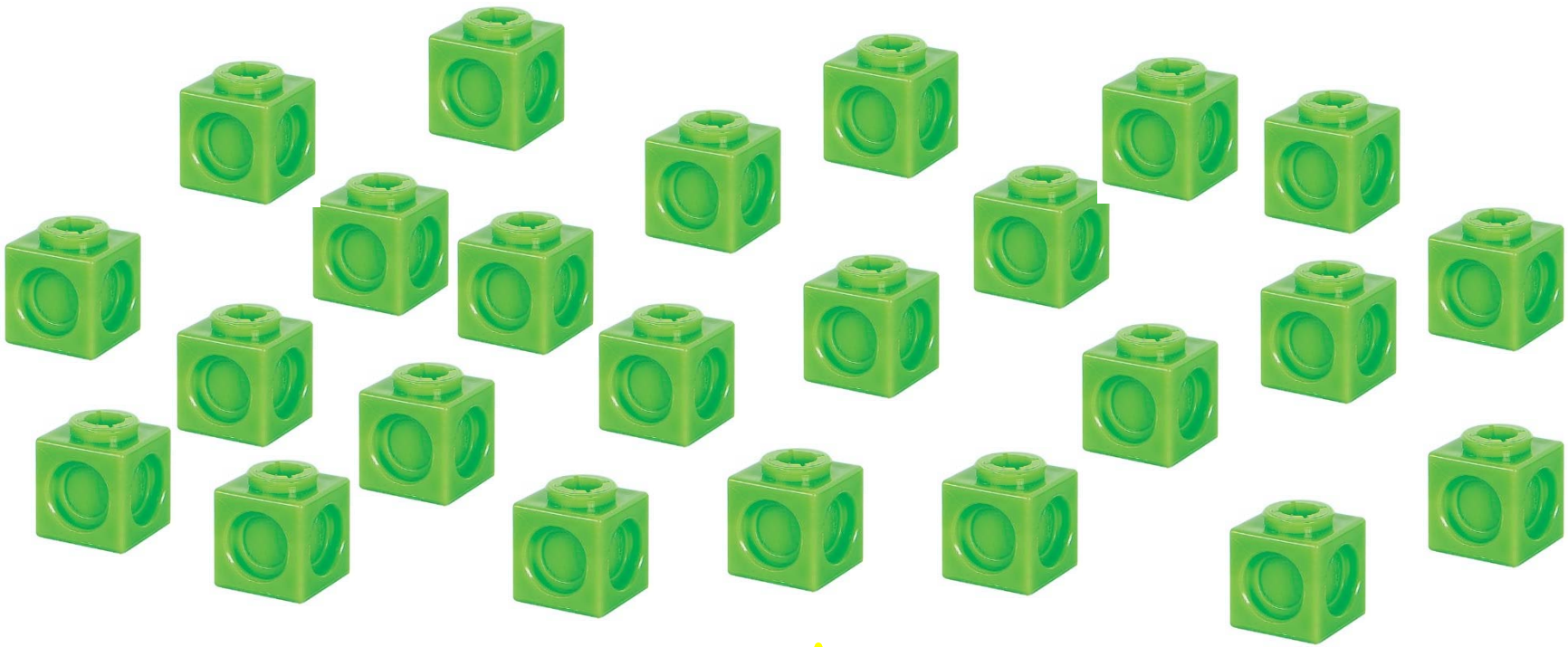
Represent one quantity as a fraction of another

Use division as an equivalent calculation

Y7U15

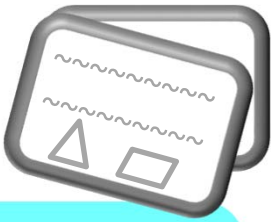
Do Now

How many ways can you divide 24 cubes into equal groups?



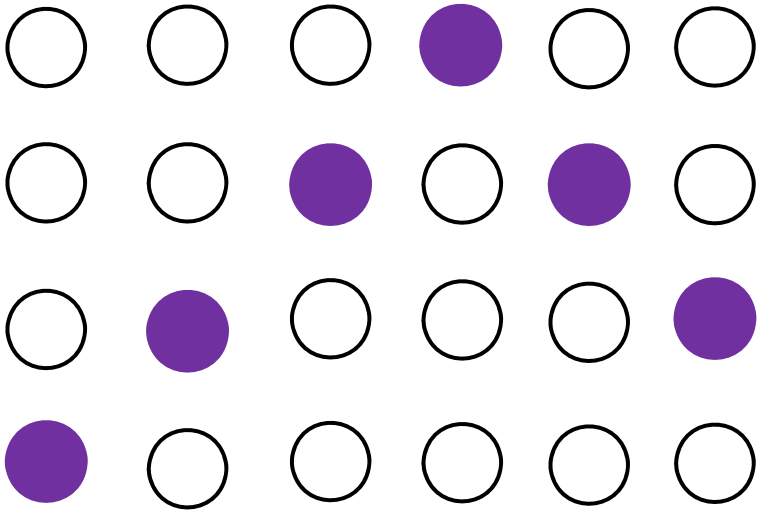
How can making different rectangles with the cubes help?

What fraction?



Fractions as part of a set.

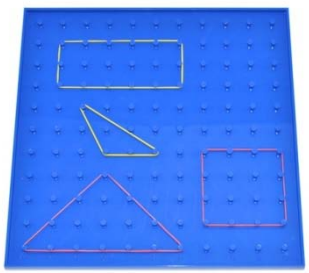
What fraction of the dots are purple?
How do you know?



Is there a quick way of seeing what **unit fraction** the purple dots represent without counting all the dots? Explain it.

Show me $\frac{1}{4}$

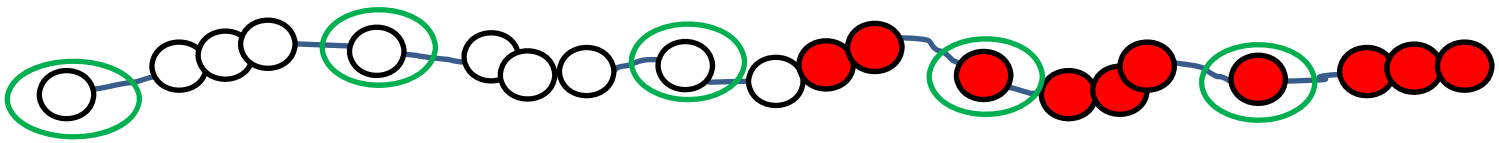
Using different manipulatives find as many ways as you can of showing $\frac{1}{4}$



How many different ways can you show $\frac{1}{4}$ with the same manipulative? How could equivalent fractions help you?

Bead string fractions

Discuss how both of these representations show $\frac{1}{4}$.

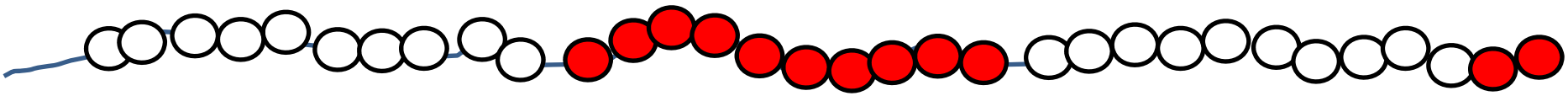


What is the same and what is different about them?

What other fractions could they be showing?

Bead string fractions

Now show $\frac{1}{4}$ using 32, 40, and 60 beads

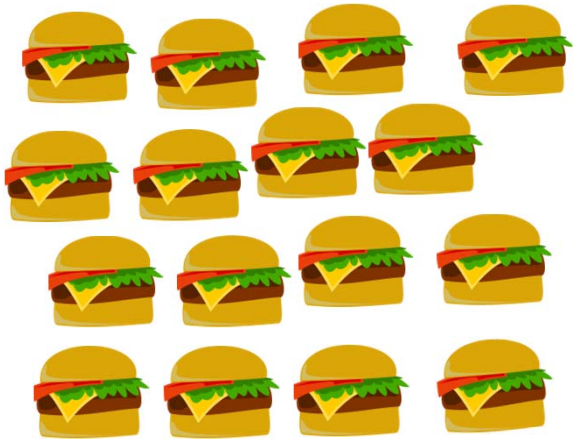


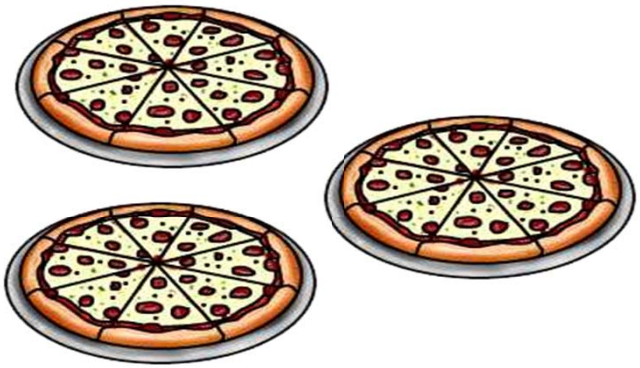
Using the same numbers of beads, show $\frac{1}{5}$

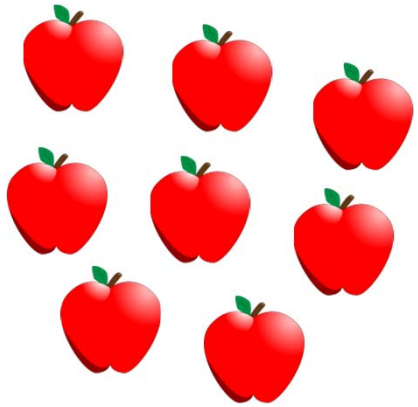
Can all of 32, 40 and 60 beads be used to show $\frac{1}{5}$? If not, why?

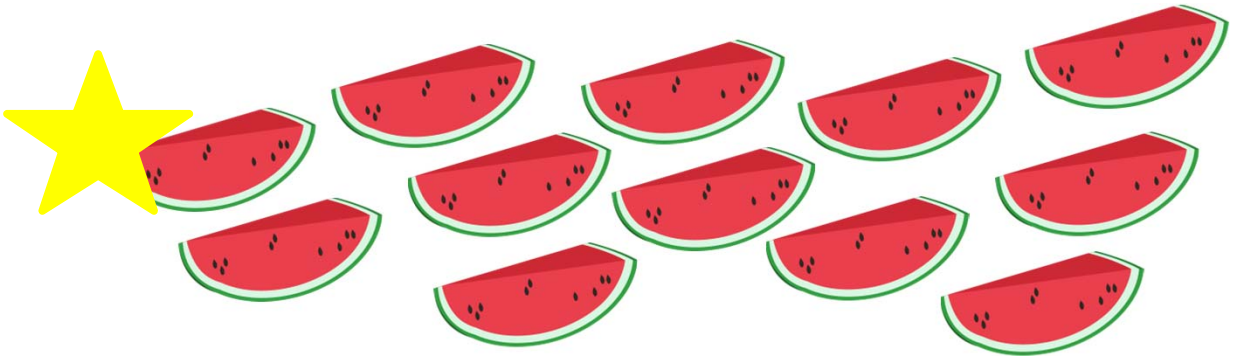
Sharing equally between 8

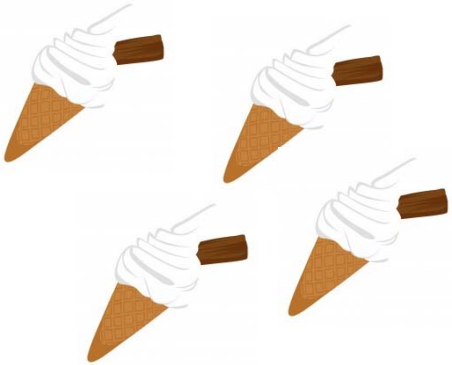
There are eight people at a picnic. How much of each food item would they get if it was shared equally between them?











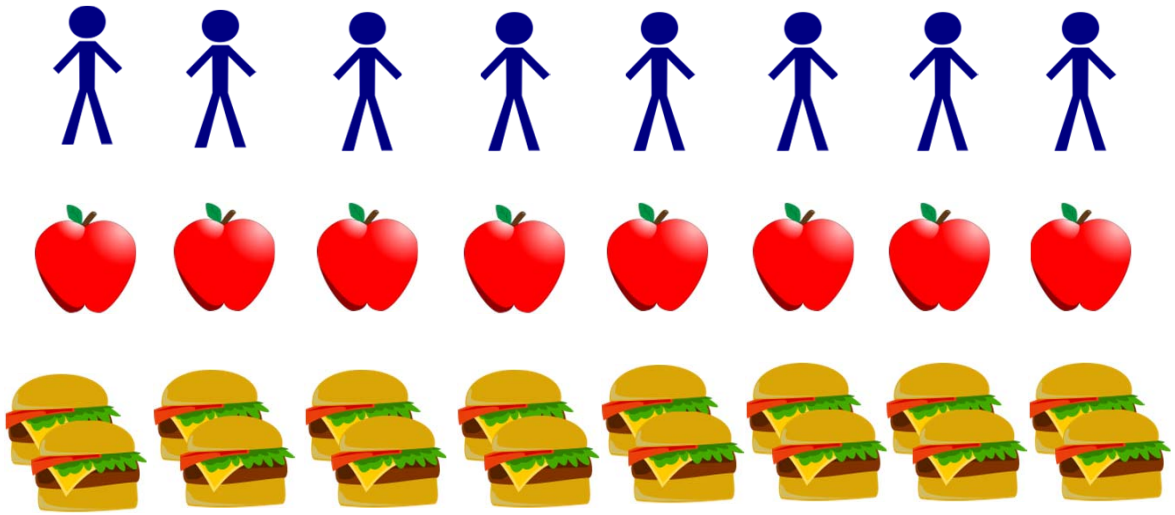
How would the quantities change if there were only 3 people?

What about if there were two people, and one ate twice as much as the other?

Sharing equally between 8

8 apples

16 burgers



One apple each

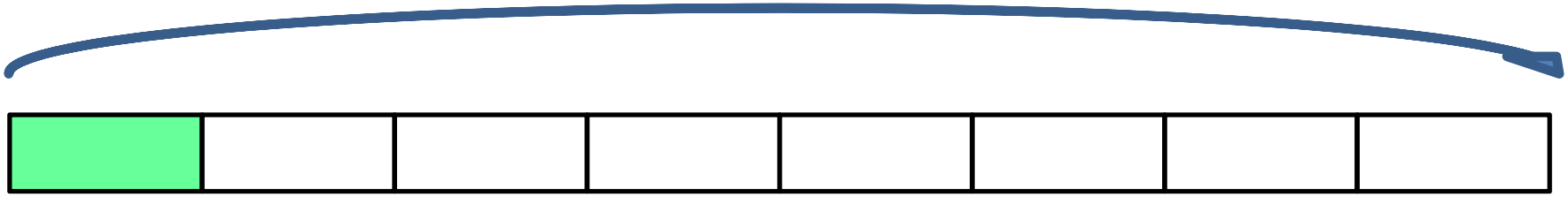
Two burgers each

What operation is helpful to share equally between 8 people?

What is an eighth of 24 pizza slices?

An eighth of 24 pizza slices

24

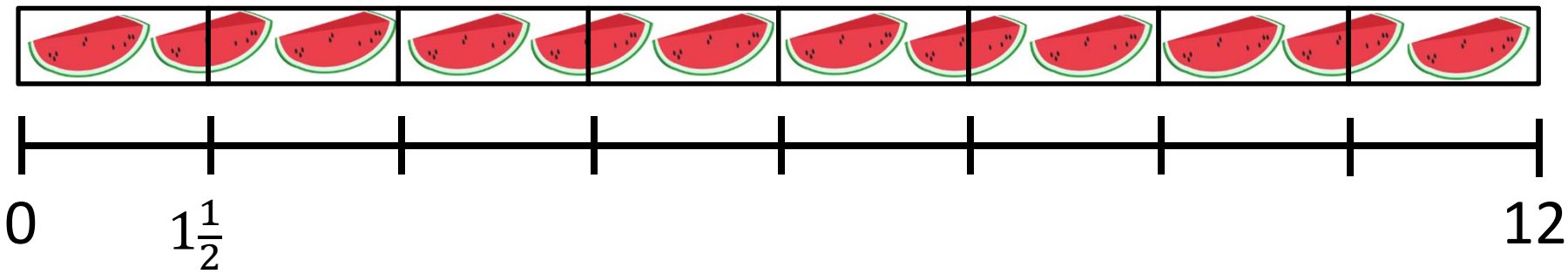


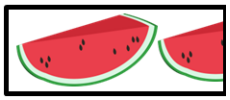
‘Divide the whole into 8 equal parts’

$$24 \div 8 = 3$$

Sharing equally between 8

How can you explain sharing 12 slices of melon between 8 people?



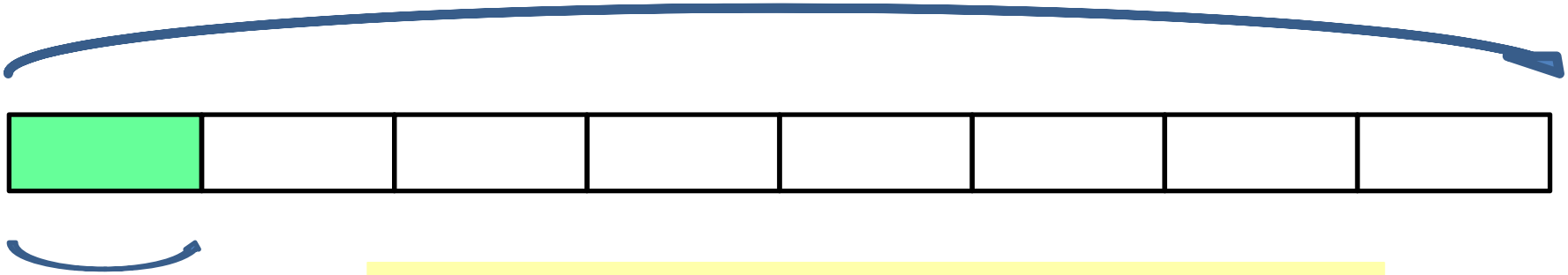


'When we shared the 12 slices between 8, each person got one whole slice and there were 4 left over, so each person got another half.'

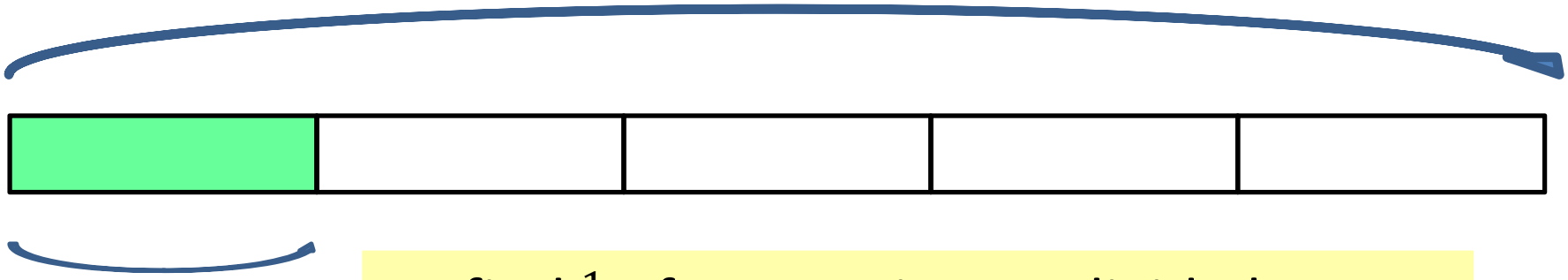
$$12 \div 8 = \frac{12}{8} = 1\frac{1}{2}$$

How else could you explain this?

Finding unit fractions of quantities



To find $\frac{1}{8}$ of a quantity we divide by 8



To find $\frac{1}{5}$ of a quantity we divide by……

To find $\frac{1}{3}$ of a quantity we divide by……

Finding unit fractions of quantities

$\frac{1}{6}$ $\frac{1}{4}$ $\frac{1}{9}$ $\frac{1}{12}$ $\frac{1}{5}$ $\frac{1}{8}$ $\frac{1}{3}$ $\frac{1}{2}$

9 24 12 10 72 3 36 48 20 15 5 96 8 4 6 30 18 54 120

Complete the sentence below using only numbers from the clouds above in as many ways as you can. Write the corresponding calculation. An example has been done for you.

_____ of _____ is _____

$\frac{1}{2}$ of 24 is 12 Calculation: $24 \div 2 = 12$

Which of the numbers in the blue cloud can be used in the middle of the sentence for every fraction? Why?

Always, sometimes or never true?

One sixth of a quantity is greater than one quarter of another quantity

Halving a quantity three times will give you one sixth of the original quantity

Calculating a quarter of an even number will always give you an even number

Provide some examples for each statement to prove that you are correct.

Create your own ‘always, sometimes or never true’ statements.

Challenge Extra

Sweet dilemma

Zak had some sweets.
He was feeling generous so gave half of his sweets to Chloe.
Chloe then gave one third of her sweets to Harry.
Harry gave one quarter of his sweets to Bella.
Bella gave one fifth of her sweets to Nicola.
Nicola received 6 sweets.

How many sweets did Zak start with?

Can you draw a bar model to show your answer?

What fraction of Zak's sweets does Nicola have?

Thursday, 04 June 2020

Lesson 2

Calculating with non-unit fractions

Key learning

Find non-unit fractions of quantities

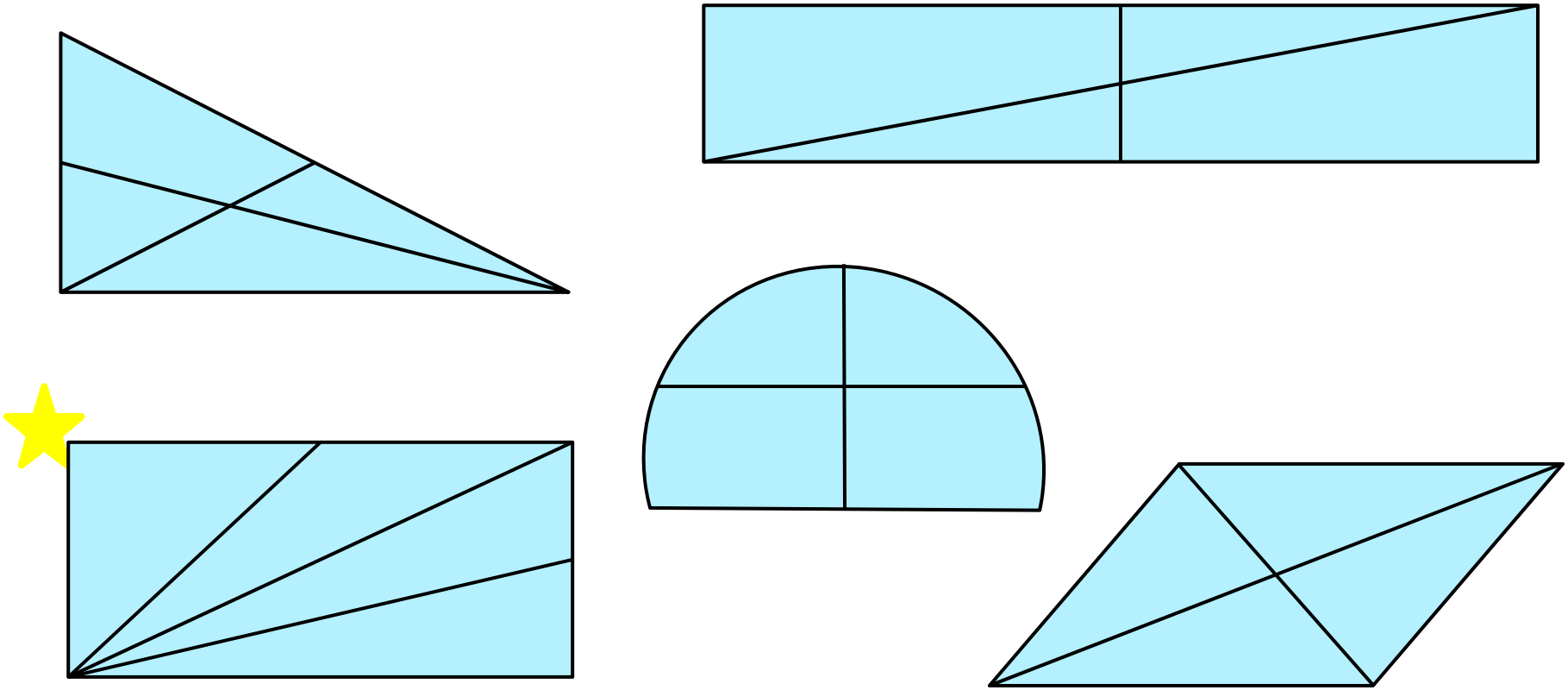
Find equivalent calculations

Represent problems using bar and line models

Y7U15

Do Now

Which of the following shapes are divided equally into quarters.
Explain how you know.



For the shapes that are not divided into quarters, for which of them can you work out what fraction of the whole each region represents?

Fact families

$\frac{1}{3}$ of 24 is 8

$\frac{1}{8}$ of 24 is 3

$3 \times 8 = 24$

Explain how multiplication or division facts can help us know what a **unit fraction** of a quantity is.

$2 \times 12 = 24$

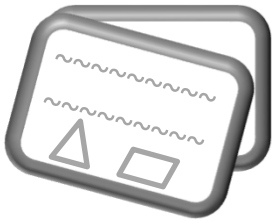
$24 \div 4 = 6$

$4 \times 9 = 36$

$36 \div 18 = 2$

For each multiplication or division fact, how many '**unit fraction** of quantity' statements is it possible to write? Is this always true? Explain why or why not.

Multilink fractions of quantities



Eight cubes represent the whole.

Show me $\frac{1}{2}$

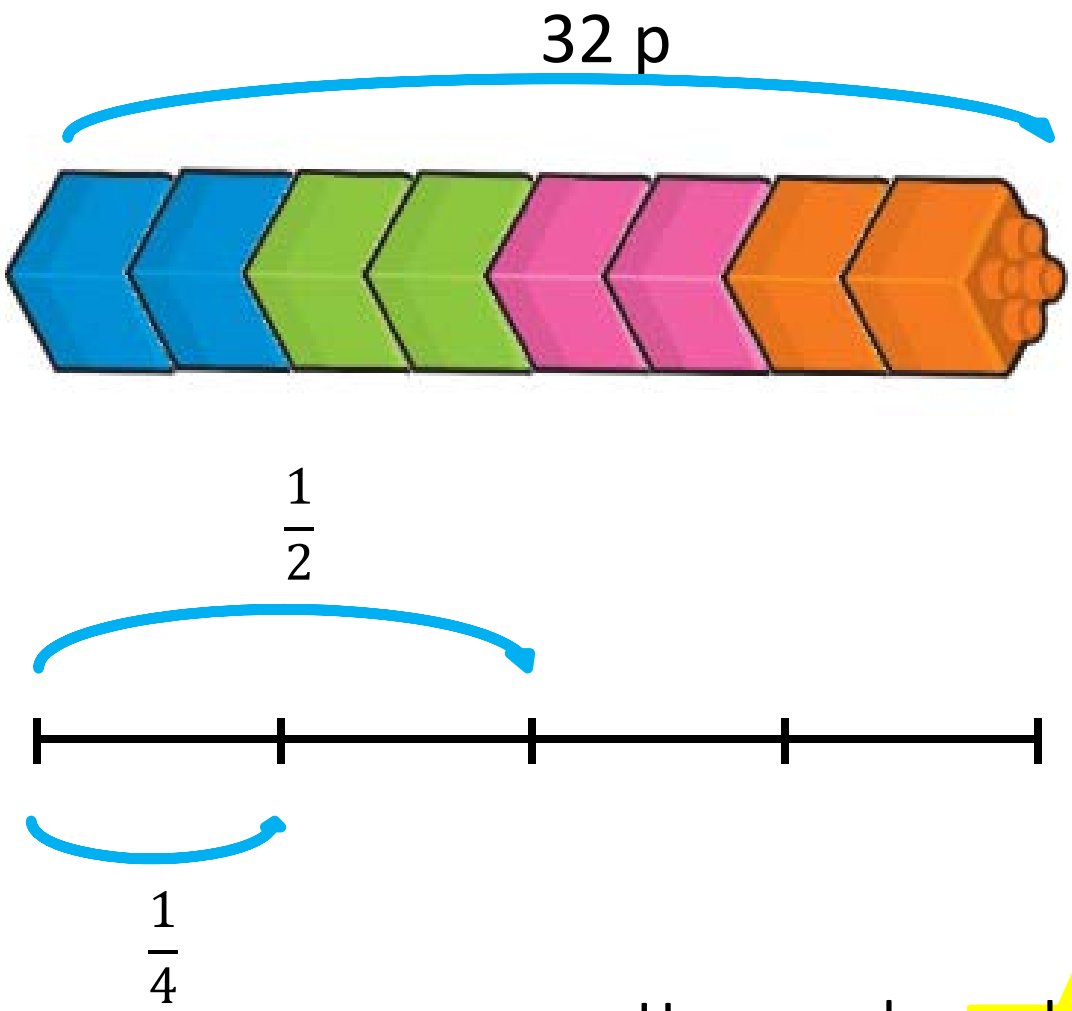
Show me $\frac{1}{4}$

Show me $\frac{3}{4}$

If the whole is worth 32 p, how much is each of the above fractions worth? How can you prove this using the cubes?

How much would $\frac{17}{16}$ be worth?

Multilink fractions of quantities



How much would $\frac{17}{16}$ be worth?

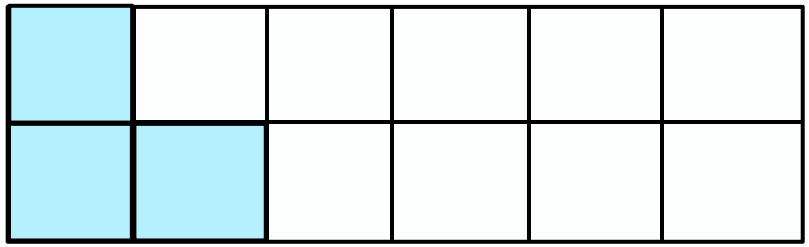
Fractions of a rectangle

Colour in $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{12}$ of the rectangle

What do you notice?

Can you find other rectangles which you can do the same with using only unit fractions?

Finding a quarter

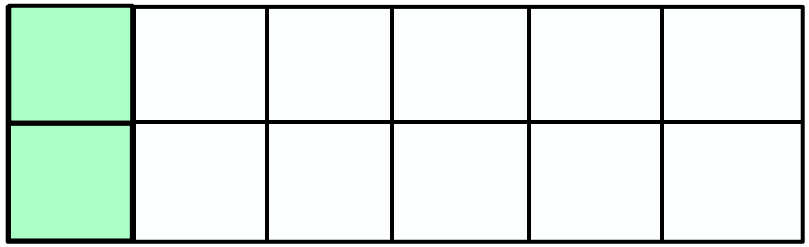


$$\frac{1}{4} \text{ of } 12 = 12 \div 4 = 3$$

What would $\frac{3}{4}$ of 12 be?

What would $\frac{5}{4}$ of 12 be?

Finding a sixth



$$\frac{1}{6} \text{ of } 12 = 12 \div 6 = 2$$

What would $\frac{5}{6}$ of 12 be?

What would $\frac{11}{6}$ of 12 be?

Fractions of a rectangle

$\frac{1}{4}$ of $12 = 12 \div 4 = 3$

What would $\frac{3}{4}$ of 12 be?

x

$\frac{1}{6}$ of $12 = 12 \div 6 = 2$

What would $\frac{5}{6}$ of 12 be?

x

Representing fractions of quantities

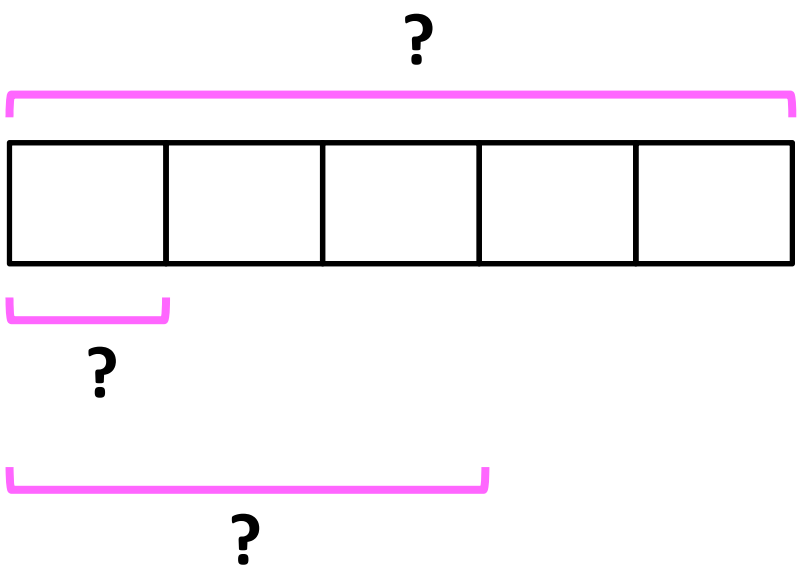
Draw a pictorial representation to help find $\frac{3}{5}$ of £90?

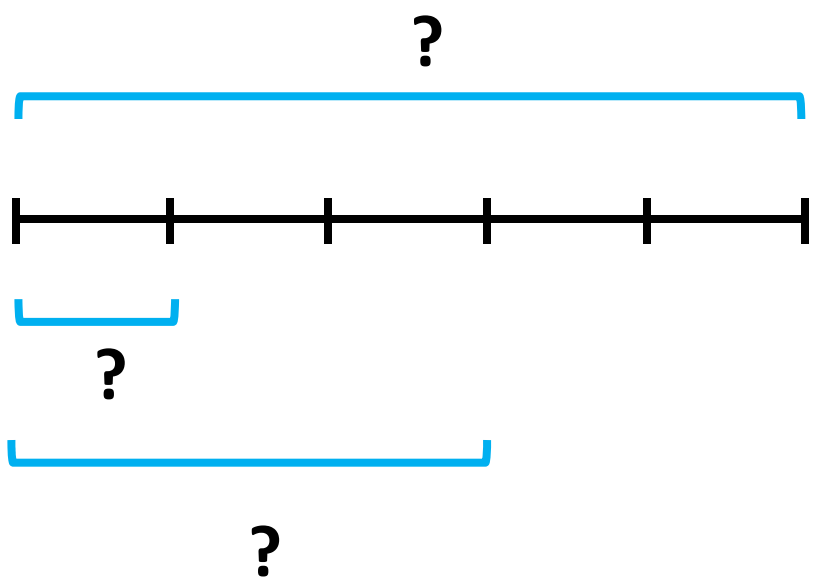
Draw a pictorial representation to help find $\frac{9}{5}$ of £30?

What is the same or different about your representations?

Representing fractions of quantities

How could you complete each model below to help find $\frac{3}{5}$ of £90?



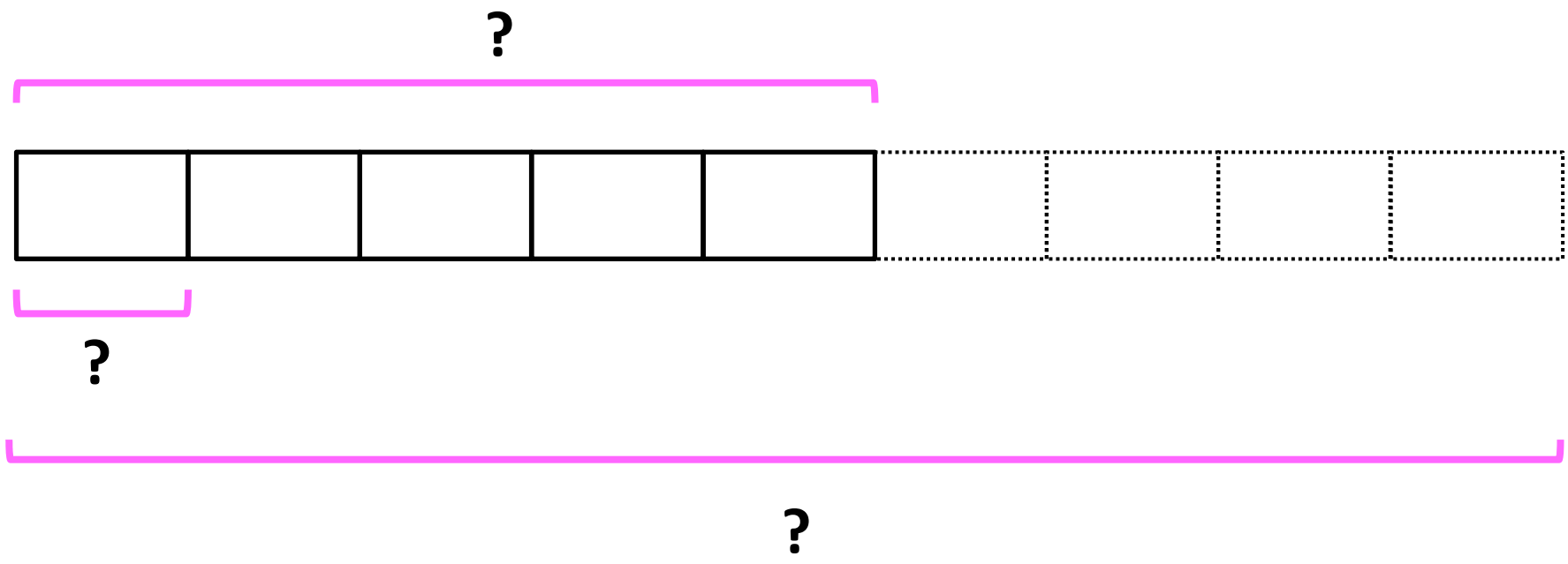


'The whole is made up of 5 equal parts'

'To find the value of each part I must divide the whole by 5'

Bar model fractions of quantities

How could you complete the bar model below to help find $\frac{9}{5}$ of £30?



'The whole is made up of 5 equal parts'

'To find the value of each part I must divide the whole by 5'

Bar model fractions of quantities

Draw bar models to help you find the solutions to these calculations.

$\frac{3}{8}$ of 32

$\frac{2}{3}$ of 96

$\frac{4}{5}$ of 555

$\frac{1}{2}$ of 19

$\frac{5}{12}$ of 1.44

$\frac{17}{10}$ of 20

$\frac{17}{5}$ of 4

$\frac{2}{5}$ of 8.6

$\frac{7}{8}$ of sixty-four hundredths

Split winnings

Jason wins his village baking competition. The prize is £240. He decides to give $\frac{1}{8}$ of the winnings to his friend Adil, $\frac{2}{5}$ to his friend Zara, and $\frac{4}{15}$ to his friend Gabby.

Put the four people in ascending order of how much they received.

Create a question....

— of $\square$ = 6

Create some questions by filling in the blanks above. Can you fill them in using:

$\frac{1}{2}$ $\frac{3}{5}$ an improper fraction a decimal

Challenge Extra

String cutting

Beth had some string of length 120 cm. Her friend Kit cut $\frac{2}{5}$ off, then another $\frac{2}{5}$ off the remaining length. Kit kept going in this way until the length of the string was less than 1 cm.

How many cuts did Kit make?

Beth thinks that if Kit continued to cut $\frac{2}{5}$ off of the length of the string, there would eventually be no string left! Is she correct?

Thursday, 04 June 2020

Lesson 3

Finding the whole given a fraction

Key learning

Given any fraction, to find the value of the whole

Given any fraction, to find the value of a different

Solve worded problems relating to finding one fraction of a quantity given another

Y7U15

Do Now

True or false?

In how many ways can you explain whether the below are true or false?

3×4 is a quarter of 12×4

3×12 is half of 6×12

4×9 is a quarter of 9×16

4×6 is a sixth of 8×18

2×7 is a third of 6×7

1×5 is a fifth of 5×10

12×18 is equal to 36×6

12×15 is a third of 18×40

Make up your own statements

One sixth is….

Each cube represents $\frac{1}{6}$ of a whole bar of gold.

Show me one whole

Show me $\frac{1}{2}$

Show me $\frac{1}{3}$

If $\frac{1}{6}$ of the whole is worth £5 000, how much would each of the fractions above be worth?
How can you prove this with your cubes?

How much would $2\frac{1}{12}$ bars be worth?

What about $2\frac{1}{4}$?

Find a whole

3 cubes is one quarter of the whole. What is the whole?

x

35 cubes is one quarter of the whole. What is the whole?

x

Find a half

3 cubes is one quarter of the whole. What is **a half**?

X

35 cubes is one quarter of the whole. What is **a half**?

X

Find a half

10 cubes is $\frac{5}{8}$ of the whole. What is **a half**?

X

40 cubes is $\frac{5}{8}$ of the whole. What is **a half**?

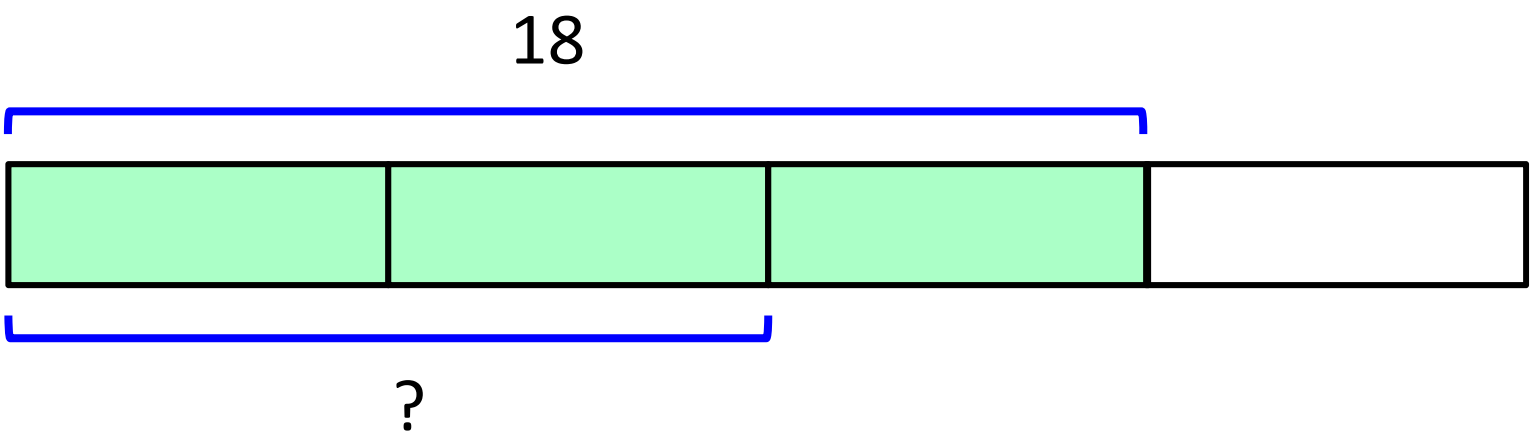
X

Find a half

Can you find a half without finding the whole first?

$\frac{3}{4}$ of a number is 18

What is a half?



What are the different ways you could calculate what a half of this number is?

Find a half

Can you find a half without finding the whole first?
Draw bar models to help you.

$\frac{5}{6}$ of a number is 35

What is a half?

$\frac{3}{8}$ of a number is 27

What is a half?

$\frac{7}{10}$ of a number is 56

What is a half?

$\frac{5}{12}$ of a number is 75

What is a half?

$\frac{9}{16}$ of a number is 108

What is a half?

$\frac{13}{20}$ of a number is 143

What is a half?

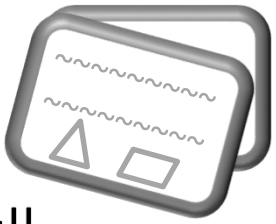
$\frac{2}{3}$ of a number is 52

What is a half?

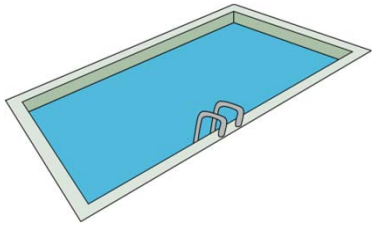
$\frac{7}{9}$ of a number is 49

What is a half?

The whole given a fraction



A swimming pool has 4 800 litres of water in it. It is only $\frac{3}{8}$ full.



Draw a bar model to show this information.

x

Use your bar model to find out the total capacity of the swimming pool.

Explain how you know what calculations to do using the bar model.

The whole given a fraction

Sam, Fabian, Abdul and Juan were in a javelin competition.
Use the information below to work out who won.
Draw bar models to explain how you calculated the full length of each throw.

$\frac{3}{5}$ of Sam's throw measured 42 m

$\frac{5}{9}$ of Fabian's throw measured 40 m

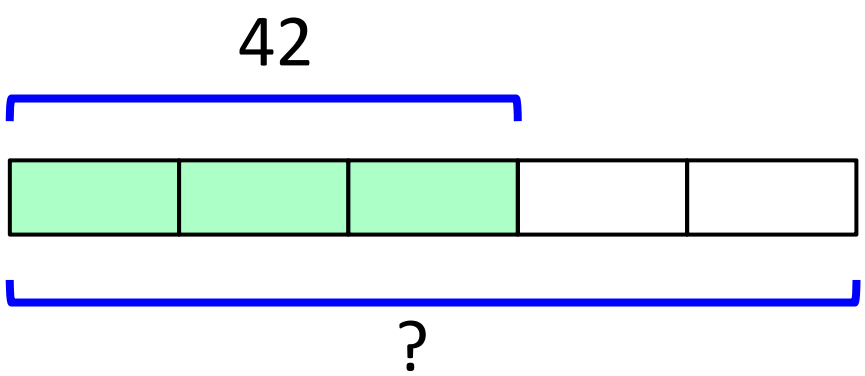
$\frac{3}{4}$ of Abdul's throw measured 51 m

$\frac{7}{15}$ of Juan's throw measured 35 m

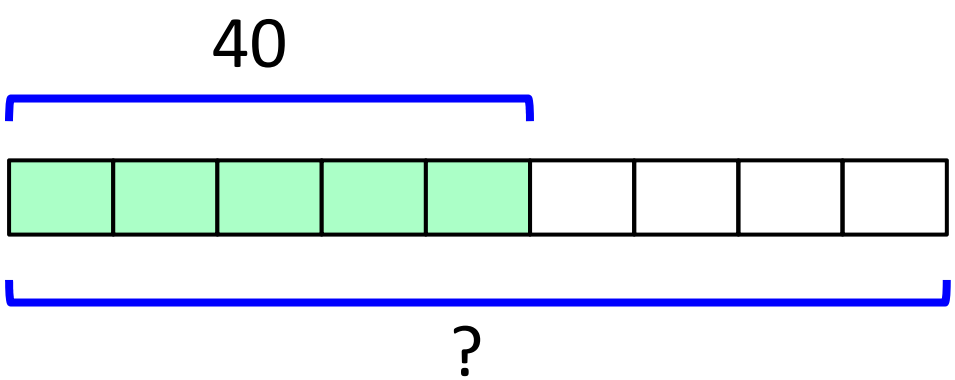
$\frac{3}{4}$ of Danny's prize was equal to $\frac{3}{10}$ of Sally's. Altogether, they won £280. How much did they each win on their own? Draw a bar model to explain your answer.

The whole given a fraction

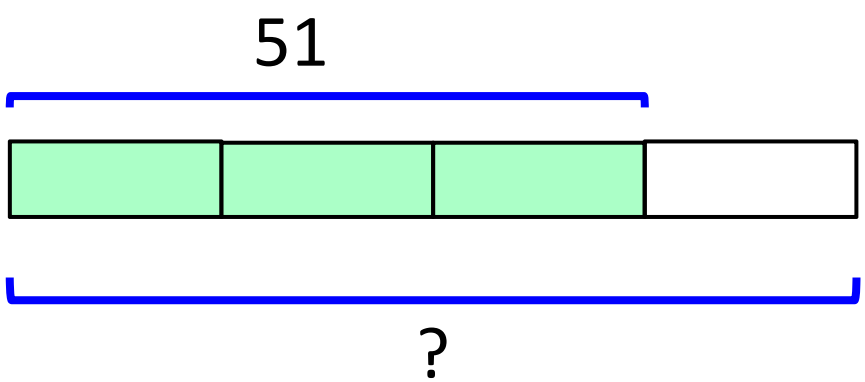
$\frac{3}{5}$ of Sam's throw measured 42 m



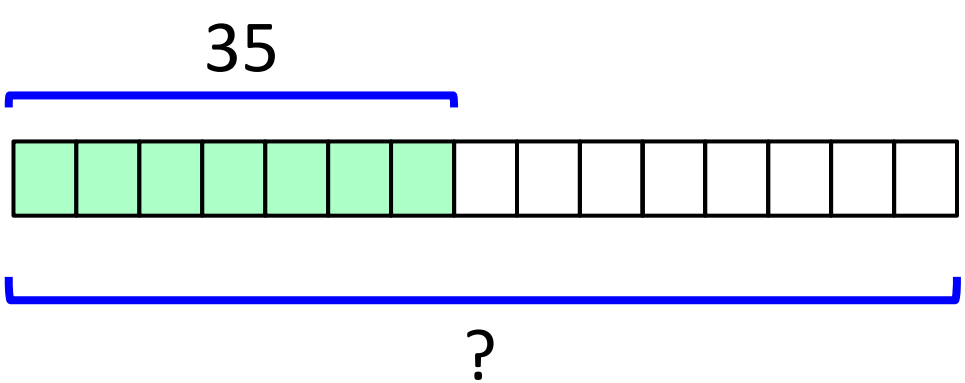
$\frac{5}{9}$ of Fabian's throw measured 40 m



$\frac{3}{4}$ of Abdul's throw measured 51 m



$\frac{7}{15}$ of Juan's throw measured 35 m



Challenge Extra

The whole given a fraction

$\frac{3}{4}$ of Danny's prize was equal to $\frac{3}{10}$ of Sally's. Altogether, they won £280. How much did they each win on their own? Draw a bar model to explain your answer.

x

Sally then gave £50 to Danny. What fraction more does Danny now have?

Thursday, 04 June 2020

Lesson 4

Fractions of quantities problems

Key learning

Find fractions of quantities

Solve problems involving fractions of quantities

Find the area of rectangles and triangles (revisiting)

Y7U15

Do Now

Find the following fractions of quantities

$\frac{5}{8}$ of 64

$\frac{2}{5}$ of 100

$\frac{5}{3}$ of 9

$\frac{1}{2}$ of 33

$\frac{12}{5}$ of 20

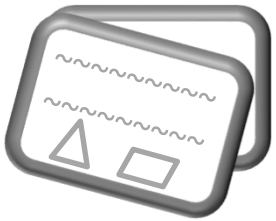
$\frac{4}{5}$ of 200

$\frac{5}{8}$ of 14.4

$\frac{4}{3}$ of 8.1

What are all the fractions of 100 that you can find without a decimal answer?

Cupcake giveaway!



Laura gave away $\frac{2}{5}$ of her cupcakes. She had 24 left.

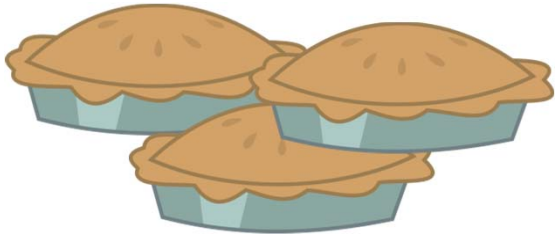
How many did she have to begin with?

Draw a bar model to help you solve this!

x

Easy as pie!

John made a batch of pies. He sold 36 of them and was left with $\frac{5}{9}$ of his original batch.



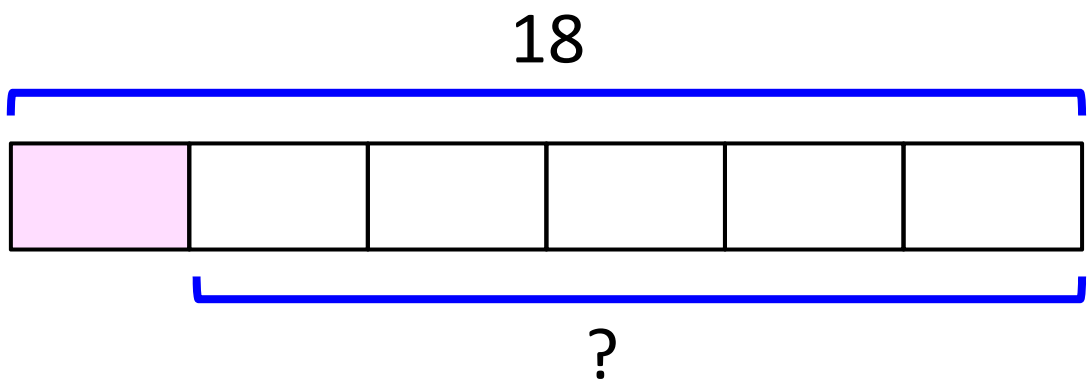
How many pies did he have left?

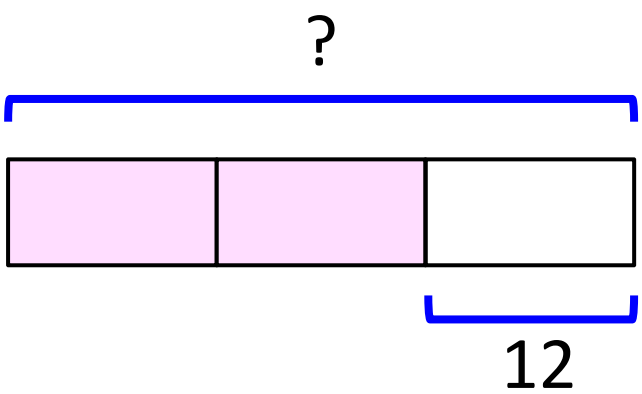
Draw a bar model to help you solve this!

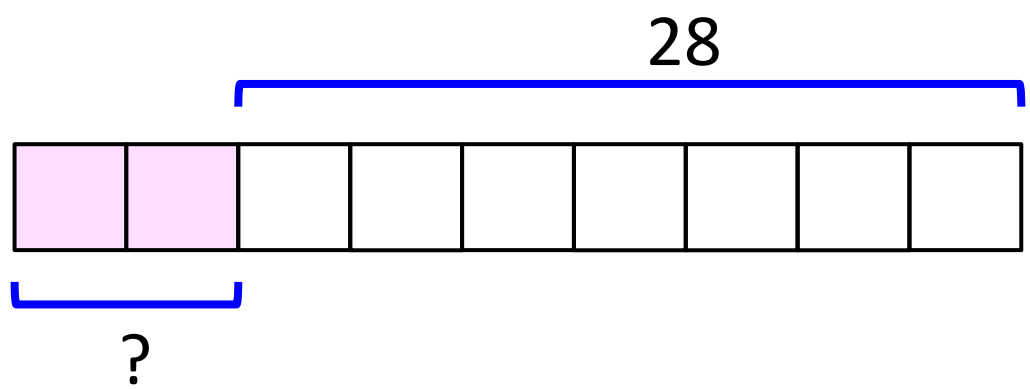
x

Making your own problems

Make your own bar modelling problems. You can use the bar models below if you want to.







Make a bar model problem where you need to draw two bars to solve it.

Sky high!

The Euclid emporium was built to be $\frac{7}{12}$ the height of Thales' tower.
Thales' tower was 232.8 m tall.

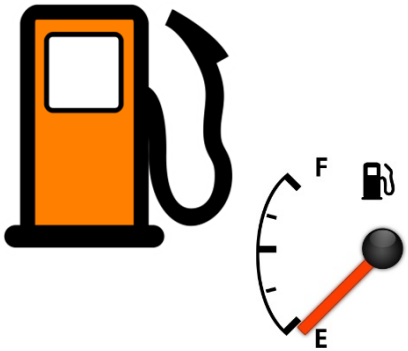
How tall was Euclid's emporium?

Draw a line model to help you solve this!

x

Out of fuel!

Charlie's car had a 60 litre fuel tank. On his journey, he had used $\frac{3}{8}$ of a full tank.



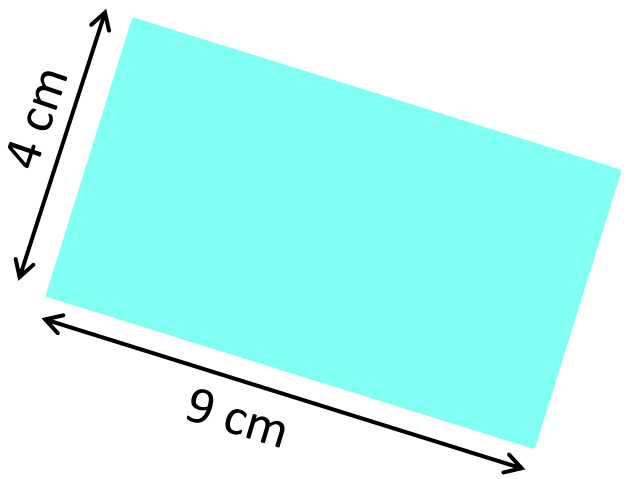
How much fuel had he used?
How much more than half a tank did he have?

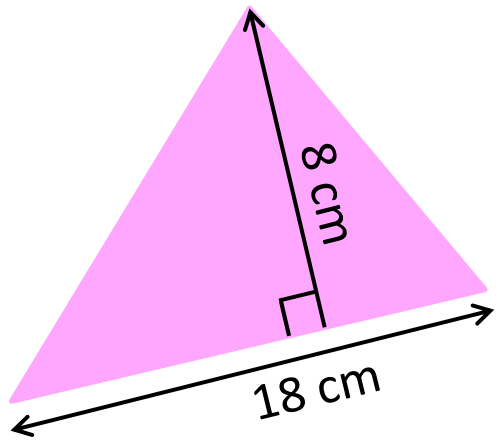
Draw a line model to help you solve this!

X

Fractional areas

How many triangles and rectangles can you make that have an area that is $\frac{2}{3}$ of the area of the rectangle and triangle below?

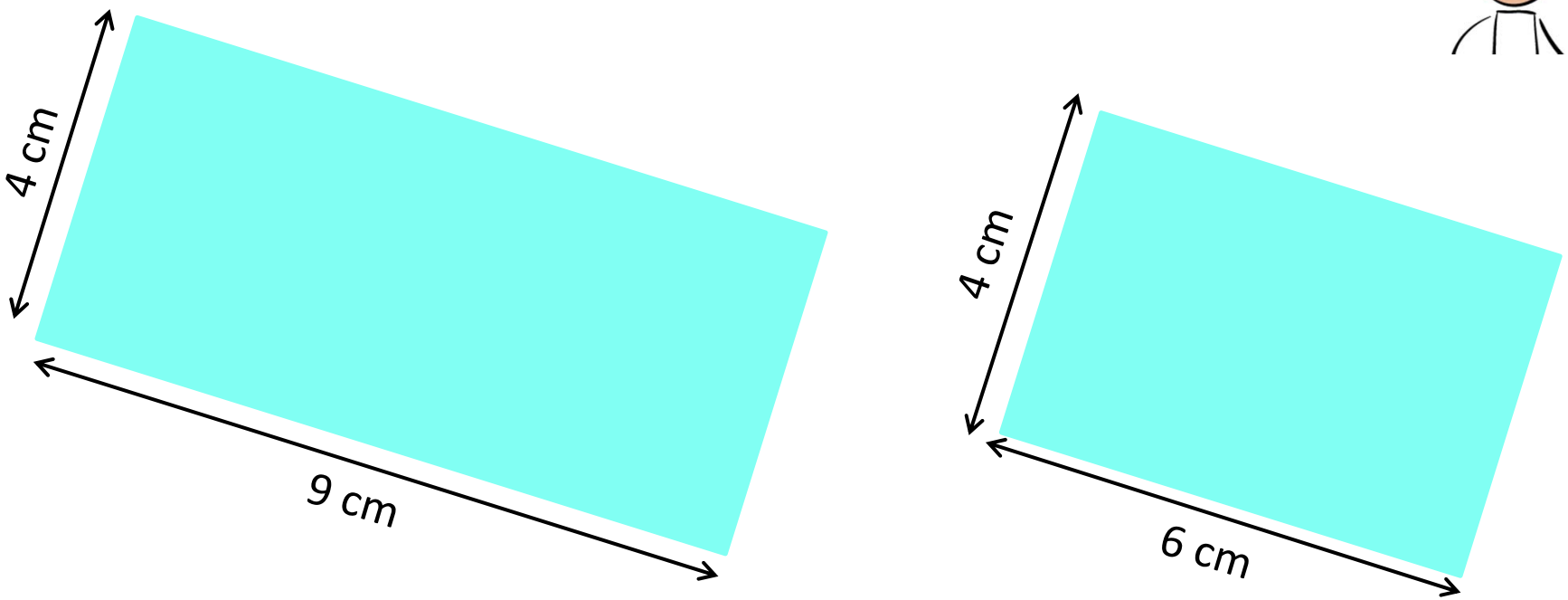




Make compound shapes have $\frac{5}{4}$ of the area of the shapes above.

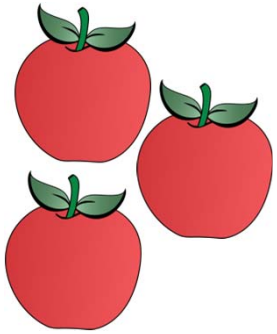
Fractional areas

Craig says to make a rectangle with $\frac{2}{3}$ the area of the blue rectangle, you just need to draw another with one length $\frac{2}{3}$ as long.



Is he correct? Explain why or why not.

Joey's apples



Joey had a sack of apples.

He dropped it and $\frac{1}{3}$ of them rolled away down the hill.

$\frac{3}{4}$ of the remaining apples had maggots in them.

He ended up with only 7 good apples.

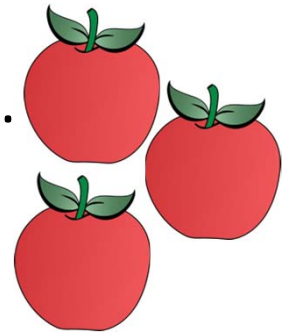
How many did he have to begin with?

Draw a bar model to help you solve this!

Joey's apples

Joey had a sack of apples.

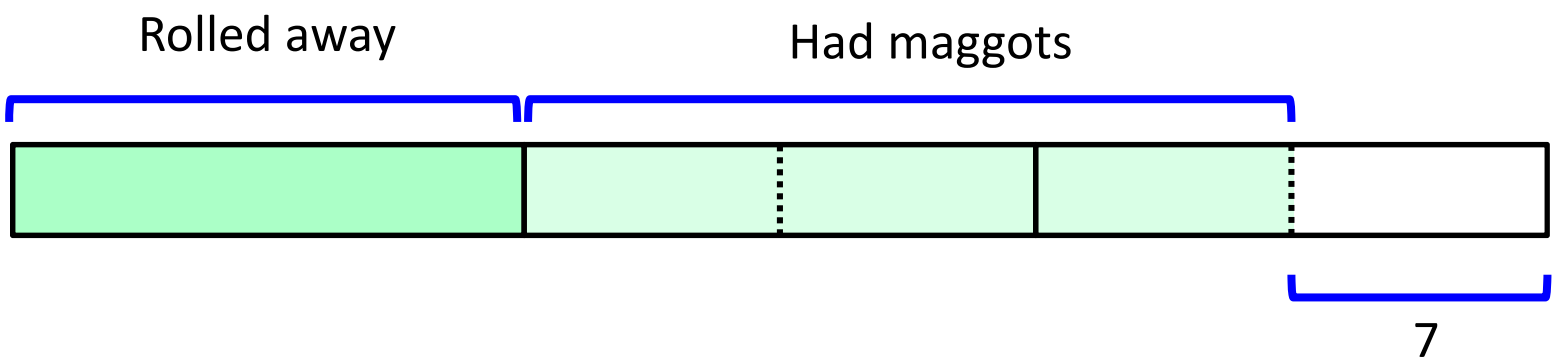
He dropped it and $\frac{1}{3}$ of them rolled away down the hill.



$\frac{3}{4}$ of the remaining apples had maggots in them.

He ended up with only 7 good apples.

How many did he have to begin with?



What fraction of the original apples were the good apples that Joey was left with?

Challenge Extra

Tip jar

Three waiters collected all the tips from a night at the restaurant in a jar.

The first waiter took $\frac{1}{3}$ of the tips.

The second waiter didn't realise the first waiter had taken his share, and took a $\frac{1}{3}$ of what remained.

The third waiter didn't realise the others had taken their shares, and also took a $\frac{1}{3}$ of what remained.

There was £16 left in the jar at the end. How much was there at the start?

Draw a bar model to help you solve this.

Challenge Extra

Taking from the tip jar

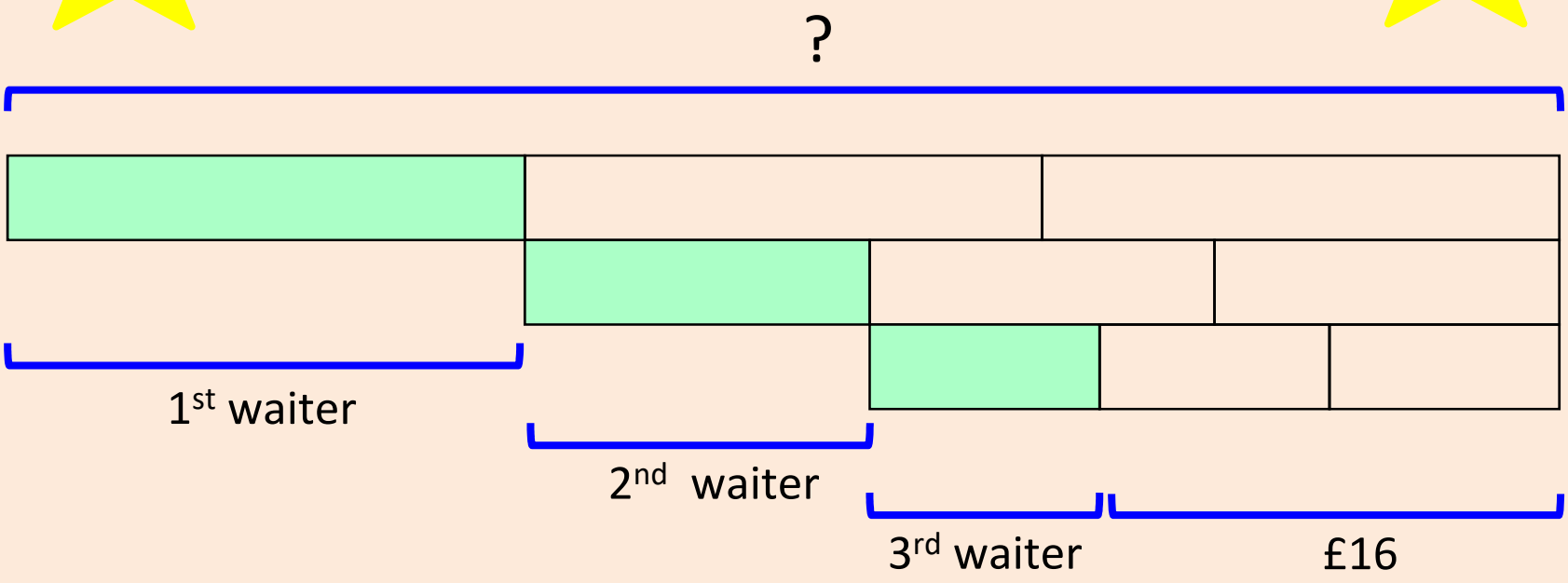
The first waiter took $\frac{1}{3}$ of the tips.

The second waiter took a $\frac{1}{3}$ of what remained.

The third waiter took a $\frac{1}{3}$ of what remained.

There was £16 left in the jar at the end. How much was there at the start?

Could you use this model to help you solve the problem?



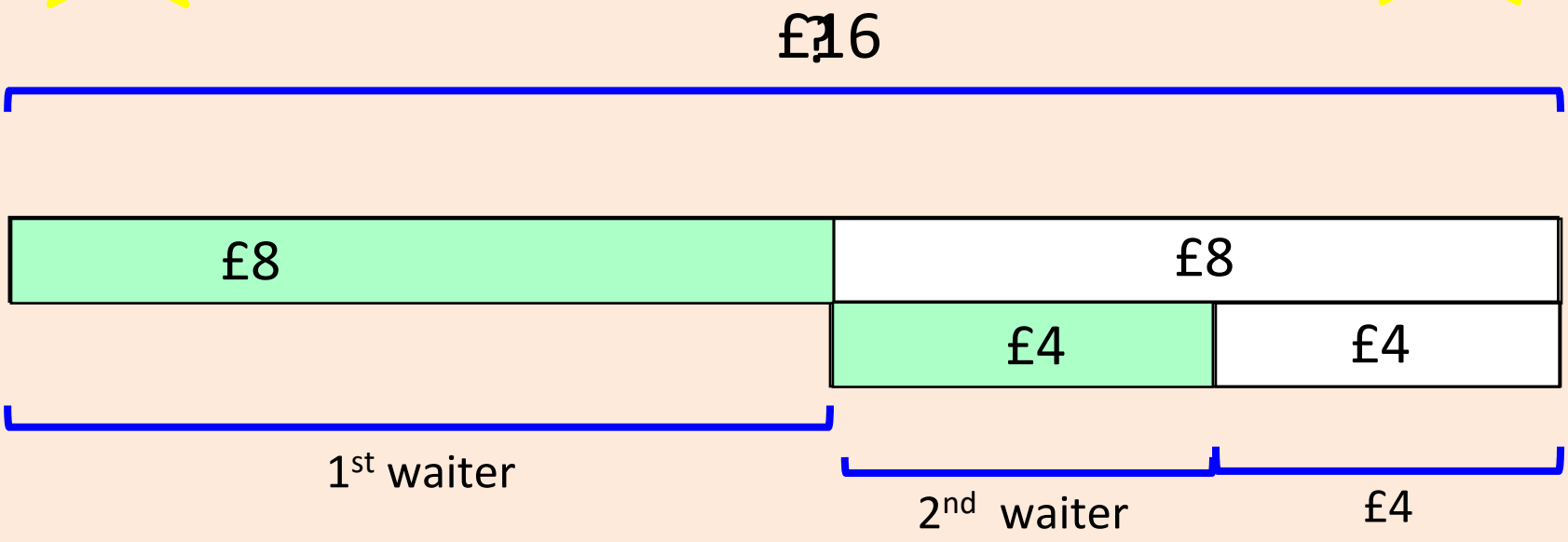
Support suggestion 1: Taking from the tip jar

The first waiter took $\frac{1}{2}$ of the tips.

The second waiter took a $\frac{1}{2}$ of what remained.

There was £4 left in the jar at the end. How much was there at the start?

Could you use this model to help you solve the problem?



Thursday, 04 June 2020

Lesson 5

Further fraction of quantity problems

Key learning

Find fractions of quantities

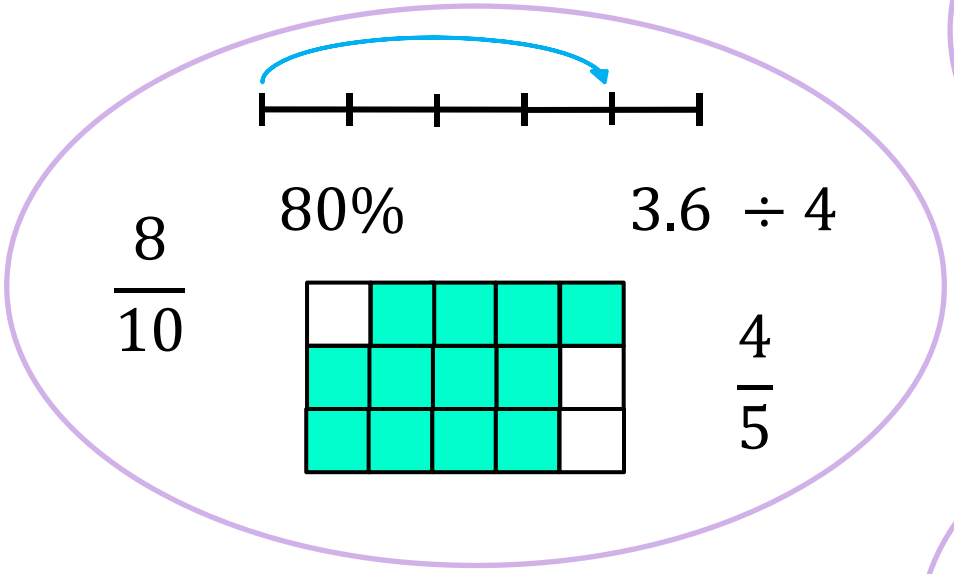
Solve problems involving fractions of quantities

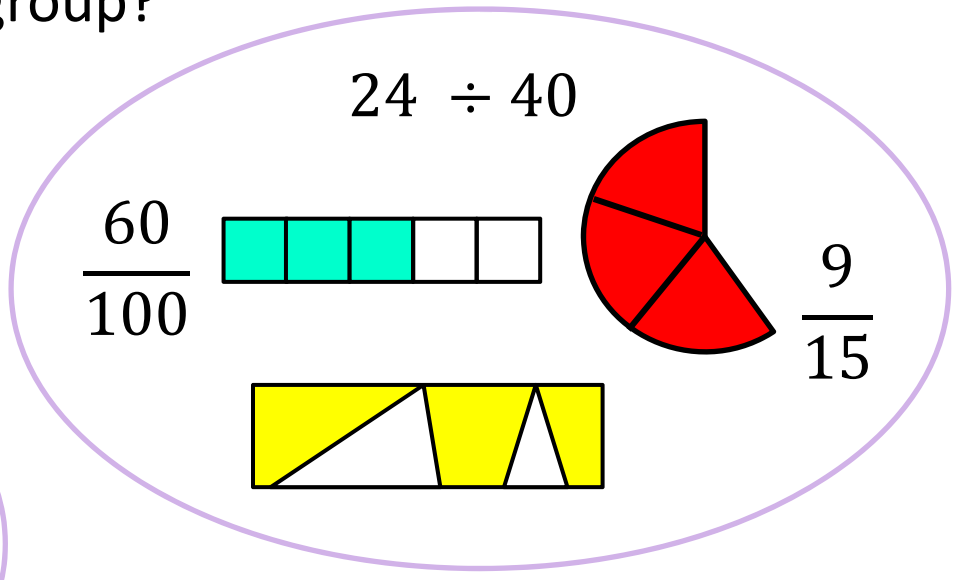
Know and use angle properties of triangles and quadrilaterals

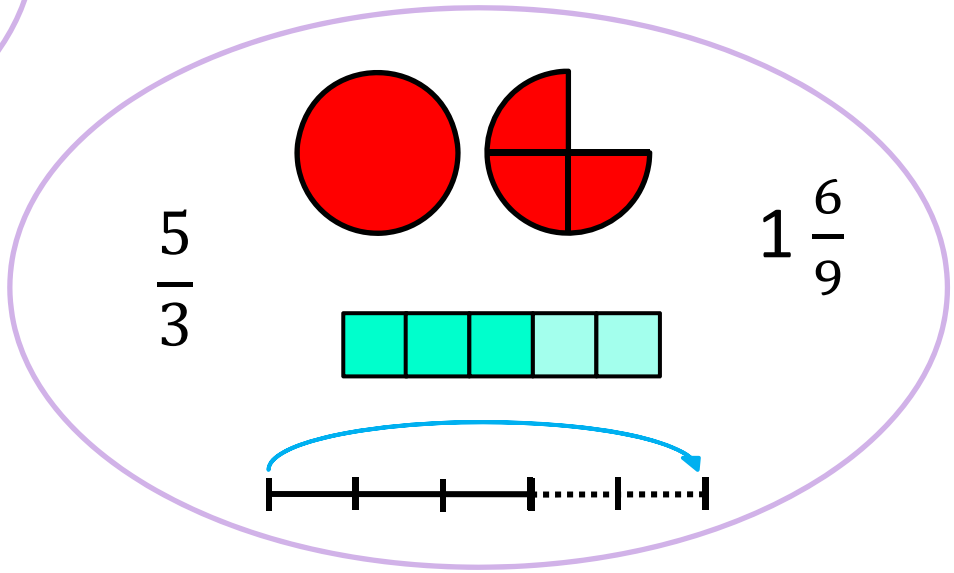
Y7U15

Do Now

Which is the odd one out in each group?







Make your own group.

True or false?

A third of the numbers are divisible by 3

Half of the numbers are even

Two fifths of the numbers are multiples of 4

A fifth of the numbers contain the digit '7'

Three tenths of the odd numbers are multiples of 3

Less than a quarter of the numbers are above 70

Nine tenths of the numbers contain two digits

Less than a third of the multiples of 3 are also multiples of 4

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

What statements could you make about the grid using the fractions below?

$\frac{1}{5}$ $\frac{7}{25}$ $\frac{19}{20}$

Do you agree?

A third of the numbers are divisible by 3

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

$\frac{33}{100}$ of the numbers are divisible by 3

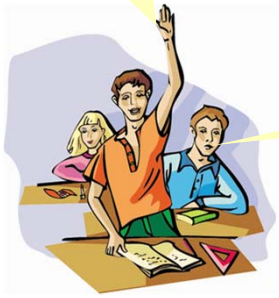
That's 33% which is a third, right?

Do you agree?

Nine tenths of the numbers contain two digits

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

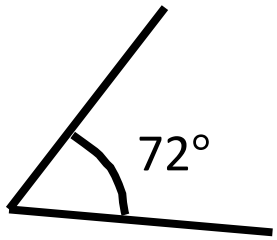
Why have you chosen all those with **less than** or **more than** two digits?



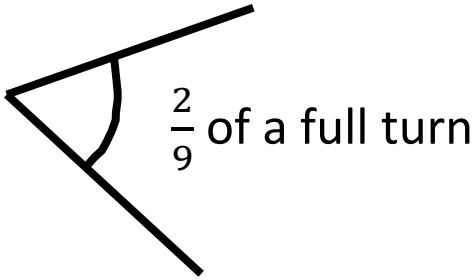
Doesn’t 100 contain two digits too?

Angle fractions

What fraction of a full turn is this angle? How do you know?

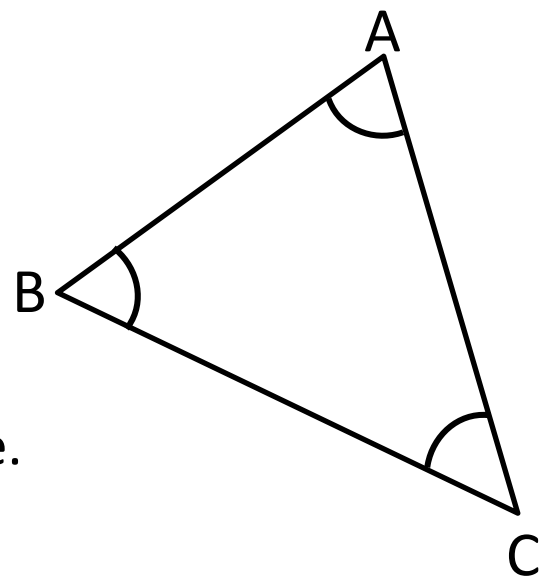


This angle is $\frac{2}{9}$ of a full turn.
How many degrees is it?



What fraction of a minute passes when a second hand on a clock rotates through 576°? How much time is this?

Angle fractions

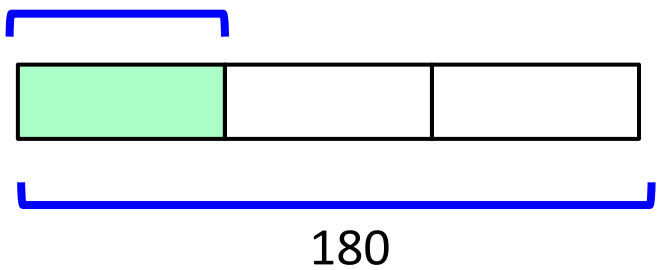


$\angle ABC$ is $\frac{1}{3}$ of the total interior angles of a triangle

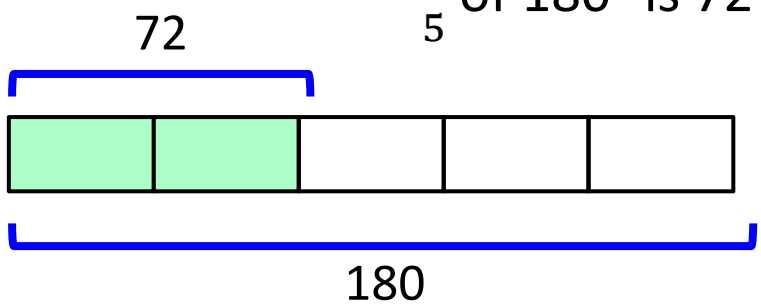
$\angle BAC$ is $\frac{2}{5}$ of the total interior angles of a triangle

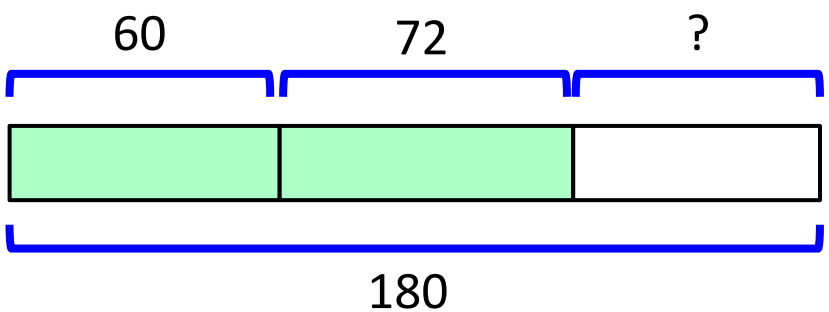
Explain what the bar models are helping to calculate.

$\frac{1}{3}$ of 180° is 60°



$\frac{2}{5}$ of 180° is 72°



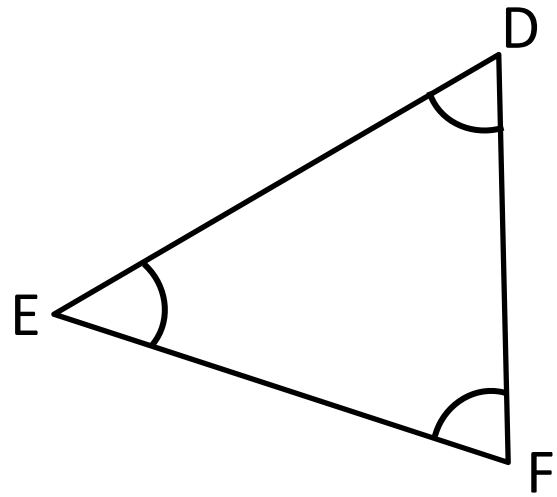


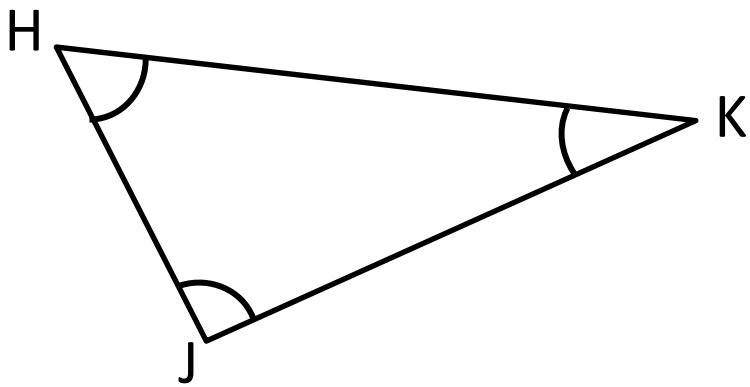
Angle fractions

Calculate all the angles on the triangles below. Give reasons for your answers.

$\angle$EDF is $\frac{1}{5}$ of the total interior angles of a triangle

$\angle$DFE is $\frac{3}{8}$ of the total interior angles of a triangle

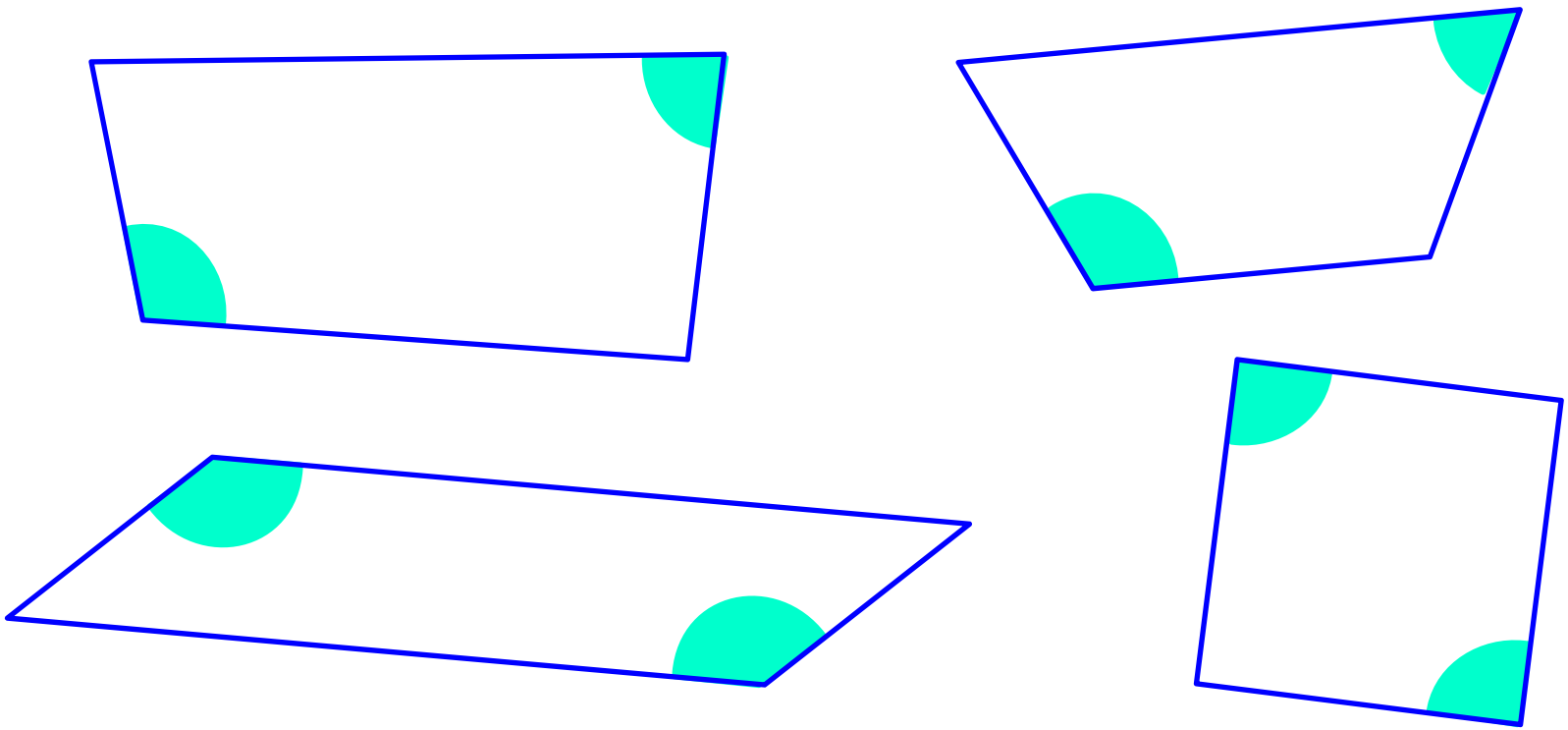




$\angle$JHK is $\frac{2}{9}$ of the total interior angles of a triangle

$\angle$HJK is $\frac{1}{3}$ of the total interior angles of a triangle

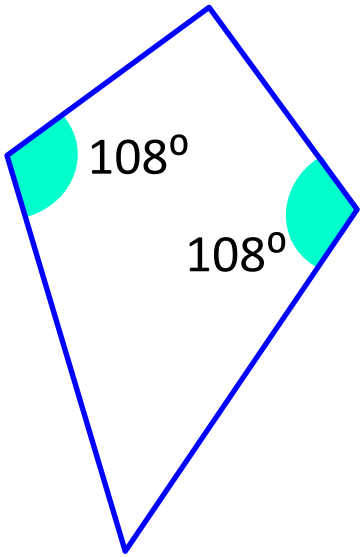
What shapes are possible?

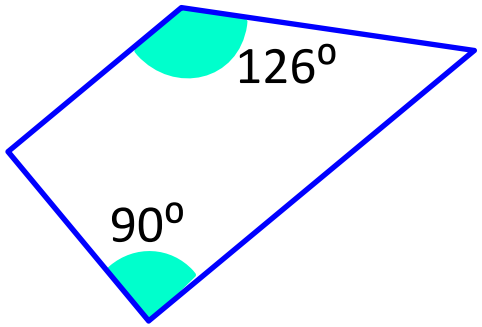


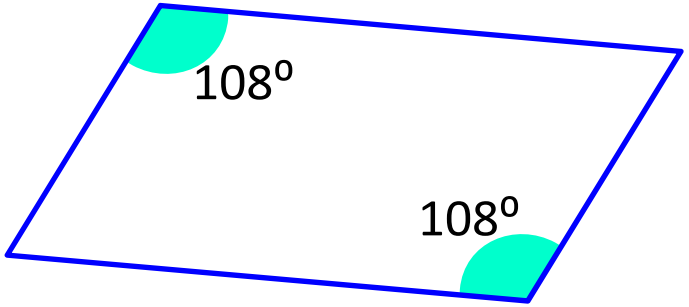
The sum of the opposite angles of a quadrilateral is $\frac{3}{5}$ of the total interior angle sum. Sketch what the quadrilateral could be.

Which different special quadrilaterals is it possible to draw?

What shapes are possible?







I can think of a special quadrilateral that we cannot draw

The opposite interior angles of the kite, trapezium and parallelogram above all add up to $\frac{3}{5}$ of the total interior angle sum

Do you agree? Explain why.

Would you rather have…

$\frac{1}{2}$ of £10 or $\frac{3}{4}$ of £8?

$\frac{3}{5}$ of £6 or $\frac{36}{60}$ of £6?

$\frac{9}{5}$ of £12 or $\frac{5}{12}$ of £30?

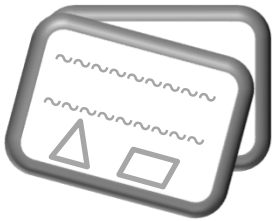
$\frac{10}{3}$ of £6 or $\frac{2}{5}$ of £40?

$\frac{7}{15}$ of £6 or $\frac{5}{9}$ of £4.50?

$\frac{11}{8}$ of £43.20 or $\frac{11}{9}$ of £46.80?

$\frac{3}{5}$ of £72 or $\frac{5}{3}$ of £26.70 or $\frac{75}{36}$ of £21.60?

What's the perimeter?



The length of the rectangle is $\frac{2}{3}$ of the width.

The total perimeter is 40 cm.

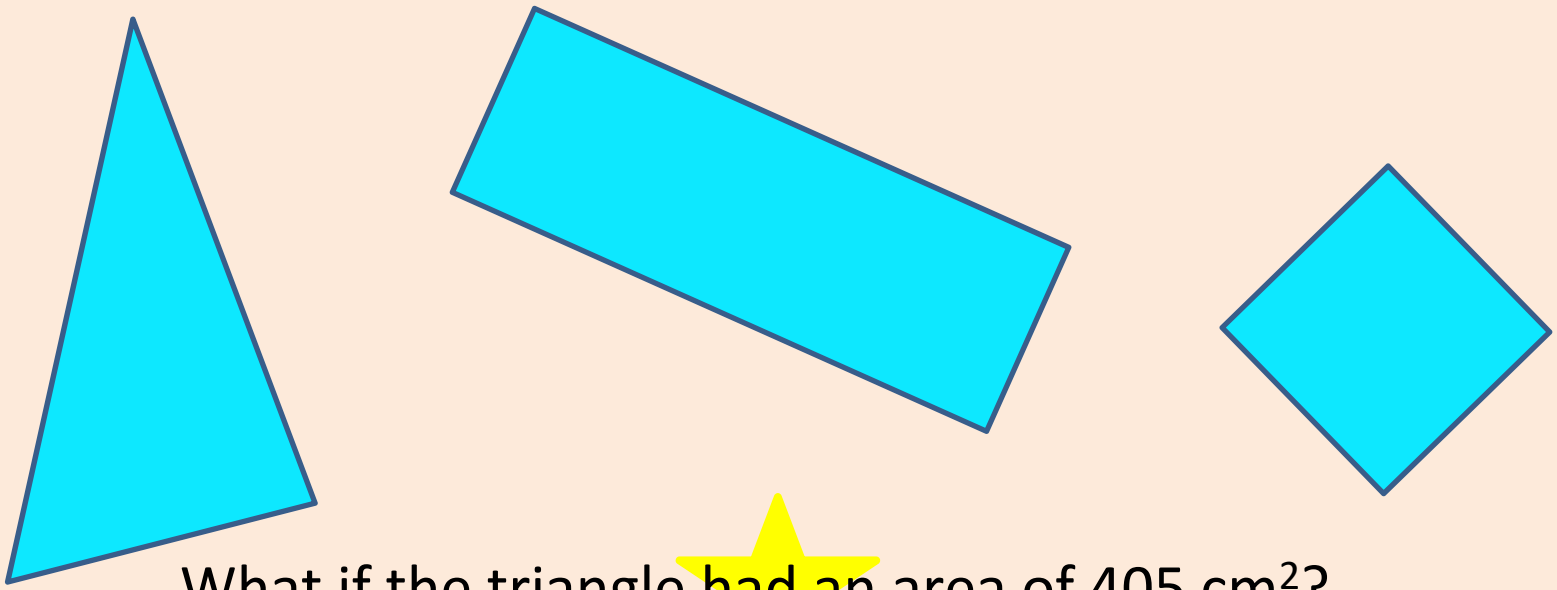
What are the dimensions of the rectangle? Draw a bar model or use cubes to help.

Challenge Extra

Triangle height

A triangle has an area of 80 cm^2. A rectangle has $\frac{3}{10}$ the area of the triangle and a square has $\frac{2}{3}$ the area of the rectangle.

If the base of the triangle is double that of the length of one side of a square, what is the triangle's height?



What if the triangle had an area of 405 cm^2?

Additional slides

These slides can be adapted and used to provide:

- Depth
- Scaffolding
- Further practice